

DETERMINATION OF TARGET THICKNESS
OF THIN LITHIUM TARGETS

C. F. Donaghy





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C. F. DONAGHY

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OF THIN LITHIUM TARGETS

by

C. F. Doraghy
Lieutenant, U. S. Navy

B.S., United States Naval Academy
(1944)

SUBMITTED IN PARTIAL FULFILLMENT OF THE
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DETERMINATION OF TARGET THICKNESS OF THIN LITHIUM TARGETS

C. F. Donaghy
Lieutenant, U. S. Navy

Submitted to the Department of Physics on May 24, 1954 in
partial fulfillment of the requirements for the degree of
Master of Science

A B S T R A C T

A study has been made of the mechanics leading to a geometric peak in the neutron yield curve of endoergic (p,n) reactions.

Theoretical expressions have been derived for the neutron yield as a function of proton energy and counter position for the case of a monoenergetic proton beam, assuming isotropic emission of neutrons in the center-of-mass system and neglecting proton straggling within the target. The derivative of the theoretical yield curve is evaluated at the peak position to get an equation for target thickness in terms of peak position, counter position, and reaction threshold. These results are applied to the $\text{Li}(p,n)$ reaction.

The effects of proton straggling and spread in the proton beam energy are then considered, and a method given to obtain an approximation to target thickness where these effects must be taken into account as is the case for very thin targets.

Thesis Supervisor: Clark Goodman
Title: Associate Professor of Physics

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3	65	M	Small intestine	Adenocarcinoma
4	70	M	Small intestine	Adenocarcinoma
5	75	M	Small intestine	Adenocarcinoma
6	80	M	Small intestine	Adenocarcinoma
7	85	M	Small intestine	Adenocarcinoma
8	90	M	Small intestine	Adenocarcinoma
9	95	M	Small intestine	Adenocarcinoma
10	100	M	Small intestine	Adenocarcinoma

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The author* takes pleasure in expressing his appreciation to Dr. Robert Kiehn for suggesting this research problem and to Professor Clark Goodman, who supervised the work while in progress, for his optimism and encouragement throughout.

The author thanks Hans Mark for the many interesting discussions of this problem and related topics over the past year.

Mrs. William Guernsey and Clyde McClelland were both of material assistance in operating the generator. The unenviable task of typing the manuscript was excellently performed by Mrs. Mary E. White in a very limited amount of time, for which the author will always be grateful.

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The author thanks him for his help in the design of this program and for his interest in the work.

Mr. William Conway and Miss Elizabeth and Paul of the National Academy of Sciences are thanked for their assistance in obtaining the manuscript. The manuscript was carefully reviewed by Mr. Mary E. White in a very limited amount of time, for which the author will always be grateful.

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I. INTRODUCTION

The $\text{Li}(p,n)$ reaction has long been a source of neutrons for experimental work. In many experiments where it is necessary to know the energy resolution of the neutrons, target thickness is the main factor contributing to this resolution, and it becomes imperative to know the value of target thickness within reasonable limits. In other experiments the neutron energy resolution may be due primarily to other effects, such as energy spread of the incident proton beam or the geometry of the experiment; since the neutron energy has an angular dependence, the finite size of materials (scatterers, absorbers, and so forth) introduces a neutron energy spread. For those experiments that fall in the latter category, as is probable when using very thin targets, an accurate determination of target thickness is not important from the viewpoint of determining the neutron energy resolution. However, even here, other considerations may require that target thickness be accurately determined.

There are many articles in the literature dealing with endo-ergic reactions in general and the $\text{Li}(p,n)$ reaction in particular, noteworthy among which is the work of Hansen, Taschek, and Williams¹. As a result of their work, they observed that by using a conventional 8-inch long counter for neutron detection, located 1 meter from the target along the beam axis, the difference between the proton energy at the geometric peak and at threshold was a good approximation of target thickness. This method of determining target thickness is

commonly used and is referred to as the "rise" method. They also discussed the use of a calibrated counter for determining the thickness of fresh lithium targets.

In the course of certain experiments at this Laboratory, a question arose concerning the accuracy of the rise method of determining target thickness, and, further, the effect of moving the counter closer to the target in order to obtain higher counting rates. This thesis is the result of a study to answer the above questions. During the course of this work, it was found that target thickness had been determined previously^{2,3} in at least two cases by fitting a theoretical yield curve to the experimental curve, but no thorough treatment of this method has been found. Such a method should give accurate results if the energy spread of the incident beam and straggling within the target are considered.

It would seem, however, that this method must be tedious, since two parameters, target thickness and effective half-angle of the counter, must be adjusted by trial and error until a best fit is obtained over the rise portion of the geometric peak. Whereas the shape of the rise portion of the curve and the height of the peak depend rather strongly on the proton energy spread within the target, the proton energy at which the peak occurs is not strongly dependent on the proton energy resolution. This situation is quite similar to the effect of resolution on the shape of a cross-section resonance.

commonly used and is referred to as "well" method. They also discussed the use of a calibrated counter for determining the thickness of fresh lithium targets.

In the course of certain experiments at this laboratory, a question arose concerning the accuracy of the thin target of beryllium target thickness, and, further, the effect of moving the counter closer to the target in order to obtain higher counting rates. This thesis is the result of a study to answer the above questions. During the course of this work, it was found that target thickness had been determined previously, in at least two cases by using a theoretical yield curve for the experimental curve, but no theoretical treatment of this method has been found. A method which gives accurate results in the energy range of the incident beam and extending within the target are considered.

It should be noted, however, that this method must be modified when two isotopes are present, target thickness and yield curve are known, the counter must be adjusted for final and given yield curve, the effect of the target on the position of the incident beam, the effect of the target on the position of the counter and the effect of the target on the position of the counter. The method is based on the fact that the counter is adjusted for final and given yield curve, the effect of the target on the position of the incident beam, the effect of the target on the position of the counter and the effect of the target on the position of the counter. The method is based on the fact that the counter is adjusted for final and given yield curve, the effect of the target on the position of the incident beam, the effect of the target on the position of the counter and the effect of the target on the position of the counter.

It is because of this that the rise method can be used for determining the target thickness in the manner indicated in this thesis.

be represented as in Figure 8, Appendix A. The reaction is pictured as occurring at the center of sphere A, the radius (V_n) representing the velocity of the neutron in the center-of-mass system. To transform to the laboratory system, it is only necessary to add the velocity of the center of mass to each point on the surface of sphere A, thus obtaining sphere B, also of radius V_n . A vector from the center of sphere A to any point on the surface of sphere B represents a particular neutron velocity (energy) in the laboratory. Thus, a group of neutrons appears in the laboratory with essentially a continuum of velocities ranging from a minimum value ($V_{cm} - V_n$) to a maximum value ($V_{cm} + V_n$), both occurring at zero degrees.

At threshold the neutrons are "squeezed" out of the nucleus with velocity (V_n) equal to zero. In this limiting case, spheres A and B are reduced to points, and all of the neutrons appear at zero degrees in the laboratory with a velocity equal to that of the center of mass. At a slightly higher proton energy, spheres A and B have a finite size as depicted in Figure 8. Here, sphere B defines a cone of neutrons of half-angle γ . At any angle less than γ , sphere B is intercepted in two points corresponding to two different neutron energies. Consequently, an element of solid angle within the neutron cone will intercept two groups of neutrons of different energies.

Since the neutron cone is defined by the angle γ for which the cone and sphere are tangent, it follows that the point of tangency corresponds to neutrons of a particular energy.

The cone gets larger with increasing proton energy until the neutron velocity in the center-of-mass system is equal to the center-of-mass velocity, at which time the cone has opened up to 2π steradians so that neutrons are being emitted into the entire forward hemisphere in the laboratory. The proton energy at which this occurs is designated E_L , and at this energy the lower-energy neutron group disappears. With any further increase of proton energy above E_L , neutrons are emitted throughout the entire 4π steradians of the laboratory.

With the long counter placed with its axis along the beam axis, the fraction (G) of neutrons emitted from thickness dE which enters the counter depends on the size of the neutron cone and the half-angle (θ) which the counter subtends at the target. If E_c is defined as that proton energy at which the neutron cone is equal to the cone subtended by the counter, then the fraction (G) has the following values (Appendix A):

$$(3a) \quad G_1 = 1 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad E < E_c$$

$$(3b) \quad G_2 = 1 - k \left(\frac{E - E_c}{E - E_T} \right)^{1/2} \quad . \quad . \quad . \quad . \quad . \quad . \quad E_c < E < E_L$$

$$(3c) \quad G_3 = 1/2 \left[1 - k \left(\frac{E - E_c}{E - E_T} \right)^{1/2} + b \left(\frac{E}{E - E_T} \right)^{1/2} \right] \quad . \quad E > E_L$$

where

$$k = \left(1 - \frac{m_1 m_3}{m_2 m_4} \sin^2 \theta \right)^{1/2} \cos \theta = \left(\frac{E_T}{E_c} \right)^{1/2} \cos \theta$$

$$b = \left(\frac{m_1 m_3}{m_2 m_4} \right)^{1/2} \sin^2 \theta$$

and $m_{1,2,3,4}$ are the masses of the projectile particle, target nucleus, resultant particle, and product nucleus, respectively.

III. CROSS SECTION

From theoretical considerations, it is shown⁴ that, for a neutron emitted with angular momentum ℓ , the cross section at energies just above threshold is given by:

$$(4) \quad \sigma_{\ell}(a,n) = \text{const } E_n^{\ell + 1/2}$$

where "a" is the charged particle inducing the reaction, and E_n is the neutron energy in the center-of-mass system. Because of the high centrifugal barrier in the region just above threshold for neutrons with angular momentum other than zero⁵, the contribution to the yield from such neutrons will be small compared with that from neutrons having zero angular momentum, hence no centrifugal barrier.

Since the threshold neutrons mostly have $\ell = 0$,

$$(5) \quad \sigma(a,n) = \text{const } E_n^{1/2} = \text{const } V_n.$$

In Appendix A, it is shown that the neutron velocity in the center-of-mass system is given by:

$$V_n = \text{const } (E - E_T)^{1/2}$$

where E_T is the threshold energy and E the instantaneous energy of the incident particle. Then the cross section σ may be written:

$$(6) \quad \sigma = C(E - E_T)^{1/2} \quad \text{where } C \text{ is a constant.}$$

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123. two hundred-forty-fourth is the fact that the two hundred-forty-fifth is the fact that the
124. two hundred-forty-sixth is the fact that the two hundred-forty-seventh is the fact that the
125. two hundred-forty-eighth is the fact that the two hundred-forty-ninth is the fact that the
126. two hundred-fiftieth is the fact that the two hundred-fifty-first is the fact that the
127. two hundred-fifty-second is the fact that the two hundred-fifty-third is the fact that the
128. two hundred-fifty-fourth is the fact that the two hundred-fifty-fifth is the fact that the
129. two hundred-fifty-sixth is the fact that the two hundred-fifty-seventh is the fact that the
130. two hundred-fifty-eighth is the fact that the two hundred-fifty-ninth is the fact that the
131. two hundred-sixtieth is the fact that the two hundred-sixty-first is the fact that the
132. two hundred-sixty-second is the fact that the two hundred-sixty-third is the fact that the
133. two hundred-sixty-fourth is the fact that the two hundred-sixty-fifth is the fact that the
134. two hundred-sixty-sixth is the fact that the two hundred-sixty-seventh is the fact that the
135. two hundred-sixty-eighth is the fact that the two hundred-sixty-ninth is the fact that the
136. two hundred-seventieth is the fact that the two hundred-seventy-first is the fact that the
137. two hundred-seventy-second is the fact that the two hundred-seventy-third is the fact that the
138. two hundred-seventy-fourth is the fact that the two hundred-seventy-fifth is the fact that the
139. two hundred-seventy-sixth is the fact that the two hundred-seventy-seventh is the fact that the
140. two hundred-seventy-eighth is the fact that the two hundred-seventy-ninth is the fact that the
141. two hundred-eightieth is the fact that the two hundred-eighty-first is the fact that the
142. two hundred-eighty-second is the fact that the two hundred-eighty-third is the fact that the
143. two hundred-eighty-fourth is the fact that the two hundred-eighty-fifth is the fact that the
144. two hundred-eighty-sixth is the fact that the two hundred-eighty-seventh is the fact that the
145. two hundred-eighty-eighth is the fact that the two hundred-eighty-ninth is the fact that the
146. two hundred-ninetieth is the fact that the two hundred-ninety-first is the fact that the
147. two hundred-ninety-second is the fact that the two hundred-ninety-third is the fact that the
148. two hundred-ninety-fourth is the fact that the two hundred-ninety-fifth is the fact that the
149. two hundred-ninety-sixth is the fact that the two hundred-ninety-seventh is the fact that the
150. two hundred-ninety-eighth is the fact that the two hundred-ninety-ninth is the fact that the
151. three hundredth is the fact that the three hundred-first is the fact that the
152. three hundred-second is the fact that the three hundred-third is the fact that the
153. three hundred-fourth is the fact that the three hundred-fifth is the fact that the
154. three hundred-sixth is the fact that the three hundred-seventh is the fact that the
155. three hundred-eighth is the fact that the three hundred-ninth is the fact that the
156. three hundred-tenth is the fact that the three hundred-eleventh is the fact that the
157. three hundred-twelfth is the fact that the three hundred-thirteenth is the fact that the
158. three hundred-fourteenth is the fact that the three hundred-fifteenth is the fact that the
159. three hundred-sixteenth is the fact that the three hundred-seventeenth is the fact that the
160. three hundred-eighteenth is the fact that the three hundred-nineteenth is the fact that the
161. three hundred-twentieth is the fact that the three hundred-twenty-first is the fact that the
162. three hundred-twenty-second is the fact that the three hundred-twenty-third is the fact that the
163. three hundred-twenty-fourth is the fact that the three hundred-twenty-fifth is the fact that the
164. three hundred-twenty-sixth is the fact that the three hundred-twenty-seventh is the fact that the
165. three hundred-twenty-eighth is the fact that the three hundred-twenty-ninth is the fact that the
166. three hundred-thirtieth is the fact that the three hundred-thirty-first is the fact that the
167. three hundred-thirty-second is the fact that the three hundred-thirty-third is the fact that the
168. three hundred-thirty-fourth is the fact that the three hundred-thirty-fifth is the fact that the
169. three hundred-thirty-sixth is the fact that the three hundred-thirty-seventh is the fact that the
170. three hundred-thirty-eighth is the fact that the three hundred-thirty-ninth is the fact that the
171. three hundred-fortieth is the fact that the three hundred-forty-first is the fact that the
172. three hundred-f

IV. COUNTER SENSITIVITY

The 8-inch long counter⁶ is the result of several attempts to find an arrangement of paraffin surrounding a boron detector such that the number of boron disintegrations is proportional to the number of primary-source neutrons and independent of their energies over a wide range. This counter consists of a paraffin cylinder 12 inches in length and 8 inches in diameter. Along its axis is a BF_3 proportional counter 1 inch in diameter and 8 inches in active length is embedded. It protrudes slightly over the front face of the paraffin but is protected from direct thermal neutrons by a cadmium shield. An aluminum tube shields the counter electrically. For insulation, the space between the counter wall and the shield is filled with ceresin wax. The central electrode consists of a Kovar wire of 10-mil diameter. The counter is filled with enriched (80 percent B^{10}) BF_3 to a pressure of 25 cm Hg. With 2700 volts applied to the wall a gas multiplication of about 10 is obtained. With the source of neutrons placed on the counter axis one meter from the face, a flat response is obtained in the region from about 0.5 to 3.0 Mev. The performance of the counter may be explained qualitatively as follows.

The length of the counter is many times greater than the mean free path of any neutrons to be detected. Neutrons entering the paraffin are degraded to thermal energies and diffuse into the detector where they give rise to $\text{B}(n,\alpha)$ reactions. Because of the large

IV. CONCLUSIONS

The first two sections of the report are devoted to a description of the experimental apparatus and the results of the measurements. In the third section the theoretical considerations are presented. The fourth section contains the conclusions and the references.

The experimental apparatus consists of a cylindrical vessel of diameter 10 cm and height 20 cm. The vessel is filled with water and a piston is placed at the bottom. The piston is connected to a lever which is pivoted at one end. The other end of the lever is connected to a scale. The scale is graduated in centimeters and millimeters. The piston is pushed down by a weight of 100 g. The displacement of the piston is measured by the scale. The results of the measurements are shown in the table below.

Weight (g)	Displacement (cm)
100	1.5
200	3.0
300	4.5
400	6.0
500	7.5
600	9.0
700	10.5
800	12.0
900	13.5
1000	15.0

The theoretical considerations are based on the assumption that the water is incompressible and that the piston is perfectly rigid. The pressure exerted by the water on the piston is equal to the weight of the water above it. The displacement of the piston is therefore proportional to the weight of the water above it. The results of the measurements are in good agreement with the theoretical predictions.

The conclusions of the experiment are that the pressure exerted by the water on the piston is proportional to the weight of the water above it. The displacement of the piston is therefore proportional to the weight of the water above it. The results of the measurements are in good agreement with the theoretical predictions.

The references are as follows:

1. "Theorie der Fluide", von L. Euler, 1762.
2. "Hydrostatica", von E. Torricelli, 1644.
3. "Mechanica", von A. Stevin, 1586.
4. "De Solidorum Incomprehensibilibus", von W. Stevin, 1630.
5. "De Solidorum Incomprehensibilibus", von W. Stevin, 1630.

cross section, the counting rate is determined essentially by the flux of thermal neutrons. For an infinitely large slab of paraffin, the efficiency would increase with increasing neutron energy, since low-energy neutrons penetrate only a short distance into the paraffin before being thermalized. Therefore, the low-energy neutrons have a better chance of escaping through the front face (instead of passing through the detector) than neutrons that were originally of higher energy and are thermalized at a greater distance from the front face. There are two reasons for this. At higher energies, more collisions are required for thermalization, and the collision cross section is smaller than at low energies. In order to minimize the dependence of the efficiency on the energy, the dimensions of the paraffin must be such that the thermalized fast neutrons have an increased chance to escape from the paraffin.

The probability that a neutron striking a long counter will be counted is a function of its energy and direction, and the distance from the counter axis at which it strikes. The conventional 8-inch long counter has been designed to give a flat response in the region from about 0.5 to 3 Mev, for a uniform distribution over the face of the counter. The situation is somewhat different in this problem as the neutrons are emitted in a cone of varying solid angle within which the energy distribution is not uniform. Only after the cone has opened up to 4π steradians is the distribution approximately uniform over the counter face, but the forward direction is still favored.

A theoretical calculation of counter sensitivity would be very difficult, if not impossible. It seems that the best solution to the problem is to consider counter sensitivity as being constant, and to use a counter which is modified to best satisfy this assumption consistent with reasonable yield.

For proton energies up to 40 kev above threshold, neutron energies will range from a few kev to about 120 kev. (Figure 13 of reference 1.) For each proton energy in this range, the counter will intercept all of the neutrons if $E < E_c$ or two groups, high and low energy, if $E > E_c$. Because of the finite thickness of the target, both situations can occur simultaneously. The net result is that neutrons of various energies are incident upon the counter for any proton energy.

The experimental sensitivity curves of Hanson and McKibben⁶ show that a 6-inch paraffin diameter gives a flatter response than the conventional 8-inch counter for neutrons in the energy range from thermal to a few hundred kev. This is explained by the fact that the sensitivity of the counter to high-energy neutrons is decreased because of the smaller mass of paraffin, and the sensitivity to low-energy neutrons is therefore relatively high. It may be inferred that a paraffin diameter less than 6 inches will give a still flatter response over the energy range of interest. Snowden and Whitehead² have used a 4-inch paraffin diameter in their work of fitting a theoretical yield curve to the experimental yield curve.

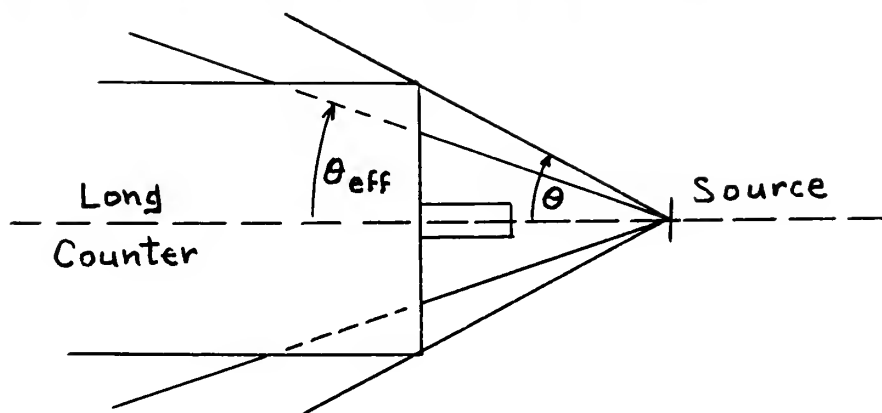
A theoretical relation of counter sensitivity to E is very difficult, if not impossible. It seems that the best solution to the problem is to consider counter sensitivity as being constant and to use a counter which is verified to best satisfy this assumption consistent with reasonable limits.

For proton energies of 10 to 100 MeV above threshold, various energies will range from a few eV to about 100 eV. (Because of reference 1.) For each proton energy in this range, the counter will intercept all of the neutrons if $E > E_c$ or the proton, high and low energy, if $E < E_c$. Because of the finite thickness of the counter, both situations can occur simultaneously. The result is that neutrons of various energies are incident upon the counter for any proton energy.

The experimental sensitivity curves of Figure 1 are obtained from data that a 6-inch paraffin detector gives a (1) constant response for conventional fission counters for neutrons in the energy range 100 eV to 100 MeV for a few hundred eV. This is explained by the fact that the sensitivity of the counter to neutrons is very low in this energy range. At the higher end of the energy range of detection, the sensitivity is very low because of the small cross section for neutron capture. It may be assumed that parallel detectors have a constant response for neutrons in the energy range 100 eV to 100 MeV. This is a reasonable assumption for the purpose of this report. The experimental results shown in Figure 1 are for a 6-inch paraffin detector.

R. A. Nobles et al⁷ have modified the shielded long counter⁶ by employing a BF_3 counter of larger diameter placed slightly farther forward in the paraffin moderator. Whereas the conventional shielded long counter shows a 10 percent drop in efficiency at 25 kev compared to the flat response region, it is claimed for the modified counter that no decrease in efficiency has occurred at 25 kev.

In the design of the optimum counter, another factor must be considered. Those neutrons which strike near the outer periphery of the paraffin have a velocity which is at an angle with respect to the counter axis. This may be represented as follows:



These neutrons have a higher probability of escaping from the counter because of the smaller amount of paraffin in the direction of their motion. The result is that the counter will have an effective half-angle (θ_{eff}) less than its true half-angle (θ). This effect is increased as the counter is moved closer to the target. It seems that

the use of a truncated cone of paraffin as employed by Bonner and Butler³ would reduce this effect. However, for optimum results, this would demand a separate truncated cone for each position of the counter. Even then, one could not expect the loss of counts to occur sharply at $E = E_c$. Possibly the best practical counter would be one having a truncated cone corresponding to the average position at which the counter is expected to be used.

The benefits of such an optimum design may not be sufficient to offset the advantage of using the conventional counter. In either case, it will be necessary to know the effective half-angle of the counter for various counter positions.

the use of a "straw" man. I will not discuss the matter here, but I will say that the "straw" man is a very common device in argument, and it is one that is often used to mislead the public. The "straw" man is a person or a group of people who are created by one side of an argument to represent the other side. The "straw" man is then attacked, and the attack is presented as if it were an attack on the other side. This is a very common device in argument, and it is one that is often used to mislead the public.

The benefit of a well-organized and efficient system is that it will be a necessary part of the company's success. It will be a necessary part of the company's success. It will be a necessary part of the company's success.

V. NEUTRON YIELD

Letting $\epsilon = \text{constant}$ and $\sigma = C(E - E_T)^{1/2}$ in accordance with the foregoing discussions, equation (2) becomes:

$$(7) \quad N_C = Z \int_{E_0 - \Delta E}^{E_0} G (E - E_T)^{1/2} dE$$

where $Z = N n f \epsilon C = \text{a constant}$, and $\Delta E = \text{target thickness}$.

Equations (3a,b,c) indicate that the form of the analytical expression defining G depends on the value of E at the point in question. This causes the integral of equation (7) to break up into two integrals if at any point within the target the proton energy has the value E_C or E_L . In order to evaluate equation (7) over the yield curve from threshold to a point beyond the geometric peak, it is necessary to know the sequence of E values. Only then can the proper G value and the limits of integration be correctly selected. Of the five possible sequences of E values discussed in Appendix B, it is there shown that only the following two sequences can occur for counter half-angles less than 30 degrees and target thicknesses less than 30 kev. (These two sequences are arranged in order of increasing E and will hereafter be referred to as cases I and II):

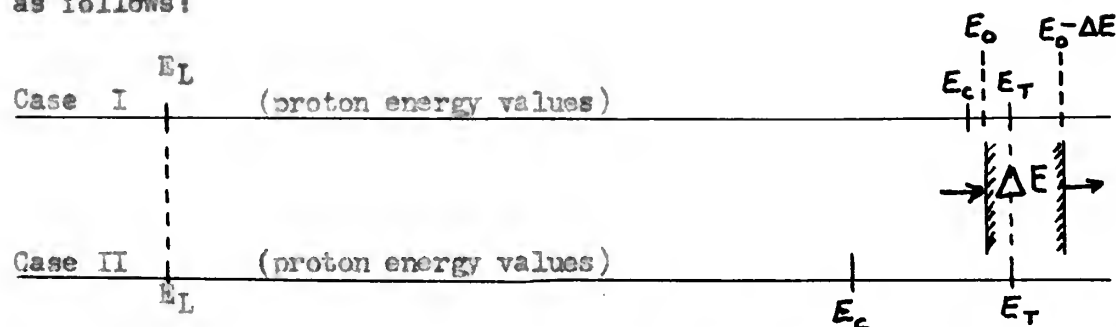
<u>I</u>	<u>II</u>
E_T	E_T
E_C	$E_T + \Delta E$
$E_T + \Delta E$	E_C
$E_C + \Delta E$	$E_C + \Delta E$
E_L	E_L
$E_L + \Delta E$	$E_L + \Delta E$

For target thicknesses up to $(E_L - E_T)/2$ (19.85 kev for lithium), case I will occur for $E_C < (E_T + \Delta E)$ and case II for $E_C > (E_T + \Delta E)$. Only case I can occur for

$$\left(\frac{E_L - E_T}{2} \right) < \Delta E < (E_L - E_C)$$

which for lithium is: 19.85 kev $< \Delta E < 30$ kev.

These two sequences may be illustrated in a typical situation as follows:



As the proton energy scales move to the right relative to the target of thickness ΔE , they indicate the manner in which the various

T	r
2	T
$4 \rightarrow 2$	4
6	$6 \rightarrow 4$
$8 \rightarrow 6$	$8 \rightarrow 6$
10	10
$12 \rightarrow 10$	$12 \rightarrow 10$

For largest displacement to be $(10) \rightarrow (12)$ we have

minimum, case 1 will occur, i.e. $(10) \rightarrow (12)$ and case 2 for

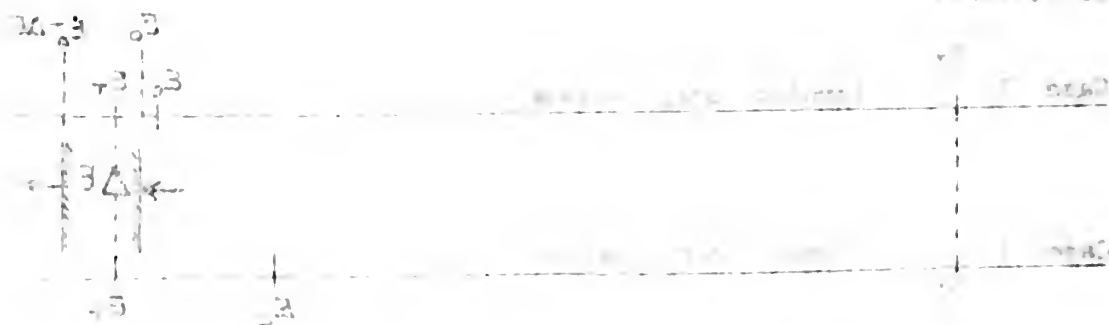
$6 \rightarrow (10) \rightarrow (12)$. If case 2 occurs then

$$(10) \rightarrow (12) \rightarrow (14) \rightarrow (16) \rightarrow (18) \rightarrow (20)$$

which for $(10) \rightarrow (12)$ is $(10) \rightarrow (12)$.

Thus the sequence is $(10) \rightarrow (12) \rightarrow (14) \rightarrow (16) \rightarrow (18) \rightarrow (20)$

as follows:



Integrating (1) with respect to x and using the condition (2) we get

the following relation (3) between y and x :

where \int is the integral

$$(3) \quad \int_{x_0}^x \frac{dx}{\sqrt{1-x^2}} = \arcsin x - \arcsin x_0$$

$$(4) \quad \int_{x_0}^x \frac{dx}{\sqrt{1-x^2}} = \arcsin x - \arcsin x_0$$

$$(5) \quad \int_{x_0}^x \frac{dx}{\sqrt{1-x^2}} = \arcsin x - \arcsin x_0$$

$$(6) \quad \int_{x_0}^x \frac{dx}{\sqrt{1-x^2}} = \arcsin x - \arcsin x_0$$

$$(7) \quad \int_{x_0}^x \frac{dx}{\sqrt{1-x^2}} = \arcsin x - \arcsin x_0$$

$$(8) \quad \int_{x_0}^x \frac{dx}{\sqrt{1-x^2}} = \arcsin x - \arcsin x_0$$

Case II: Nc/Z equals

$$(9a) \int_{E_T}^{E_0} G_1 \sigma \, dE \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad E_T < E_0 < E_T + \Delta E$$

$$(9b) \int_{E_0 - \Delta E}^{E_0} G_1 \sigma \, dE \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad E_T + \Delta E < E_0 < E_C$$

$$(9c) \int_{E_0 - \Delta E}^{E_C} G_1 \sigma \, dE + \int_{E_C}^{E_0} G_2 \sigma \, dE \quad . \quad . \quad . \quad E_C < E_0 < E_C + \Delta E$$

(9d,e,f) These are identical to (8d,e,f), respectively.

It is seen from equations (3a,b,c) that $G \sigma$ has the values:

$$G_1 \sigma = (E - E_T)^{1/2}$$

$$G_2 \sigma = (E - E_T)^{1/2} - k(E - E_C)^{1/2} = G_1 \sigma - k(E - E_C)^{1/2}$$

$$G_3 \sigma = 1/2 (E - E_T)^{1/2} - k(E - E_C)^{1/2} + b E^{1/2}$$

$$= 1/2 G_2 \sigma + b E^{1/2}$$

Substituting these expressions for $G_{1,2,3} \sigma$ in equations (8) and (9) gives:

Case II: $\alpha \in \mathbb{C} \setminus \mathbb{R}$

(2a) $\int_{\gamma} \frac{f(z)}{z - \alpha} dz = \dots$

(2b) $\int_{\gamma} \frac{f(z)}{z - \alpha} dz = \dots$

(2c) $\int_{\gamma} \frac{f(z)}{z - \alpha} dz = \dots$

(2d) $\int_{\gamma} \frac{f(z)}{z - \alpha} dz = \dots$

It is easy to see that $\int_{\gamma} \frac{f(z)}{z - \alpha} dz = \dots$

where

$$\int_{\gamma} \frac{f(z)}{z - \alpha} dz = \dots$$

$$\int_{\gamma} \frac{f(z)}{z - \alpha} dz = \dots$$

$$\int_{\gamma} \frac{f(z)}{z - \alpha} dz = \dots$$

$$\int_{\gamma} \frac{f(z)}{z - \alpha} dz = \dots$$

where γ is a path in the complex plane.

(2e) $\int_{\gamma} \frac{f(z)}{z - \alpha} dz = \dots$

Case I: Nc/Z equals

$$(10a) \int_{E_T}^{E_0} (E - E_T)^{1/2} dE \quad . \quad . \quad . \quad . \quad . \quad . \quad E_T < E_0 < E_C$$

$$(10b) \int_{E_T}^{E_0} (E - E_T)^{1/2} dE - k \int_{E_C}^{E_0} (E - E_C)^{1/2} dE \quad . \quad E_C < E_0 < E_T + \Delta E$$

$$(10c) \int_{E_0 - \Delta E}^{E_0} (E - E_T)^{1/2} dE - k \int_{E_C}^{E_0} (E - E_C)^{1/2} dE \quad . \quad E_T + \Delta E < E_0 < E_C + \Delta E$$

$$(10d) \int_{E_0 - \Delta E}^{E_0} \left[(E - E_T)^{1/2} - k(E - E_C)^{1/2} \right] dE \quad . \quad . \quad . \quad . \quad E_C + \Delta E < E_0 < E_L$$

$$(10e) \int_{E_0 - \Delta E}^{E_0} \left[(E - E_T)^{1/2} - k(E - E_C)^{1/2} \right] dE - 1/2 \int_{E_L}^{E_0} \left[(E - E_T)^{1/2} \right.$$

$$\left. - k(E - E_C)^{1/2} - b E^{1/2} \right] dE \quad . \quad . \quad . \quad E_L < E_0 < E_L + \Delta E$$

$$(10f) 1/2 \int_{E_0 - \Delta E}^{E_0} \left[(E - E_T)^{1/2} - k(E - E_C)^{1/2} + b E^{1/2} \right] dE \quad E_0 > E_L + \Delta E$$

Case 1: $\alpha \neq 0$

$$(100) \quad \int_{\alpha}^{\beta} (x - \alpha)^{-1} dx = \dots > \dots$$

$$(101) \quad \int_{\alpha}^{\beta} (x - \alpha)^{-1} dx = \dots > \dots$$

$$(102) \quad \int_{\alpha}^{\beta} (x - \alpha)^{-1} dx = \dots > \dots$$

$$(103) \quad \int_{\alpha}^{\beta} (x - \alpha)^{-1} dx = \dots > \dots$$

$$(104) \quad \int_{\alpha}^{\beta} (x - \alpha)^{-1} dx = \dots > \dots$$

$$(105) \quad \int_{\alpha}^{\beta} (x - \alpha)^{-1} dx = \dots > \dots$$

$$(106) \quad \int_{\alpha}^{\beta} (x - \alpha)^{-1} dx = \dots > \dots$$

Case II: N_c/Z equals

$$(11a) \int_{E_T}^{E_0} (E - E_T)^{1/2} dE \quad . \quad . \quad . \quad . \quad . \quad . \quad E_T < E_0 < E_T + \Delta E$$

$$(11b) \int_{E_0 - \Delta E}^{E_0} (E - E_T)^{1/2} dE \quad . \quad . \quad . \quad . \quad . \quad . \quad E_T + \Delta E < E_0 < E_c$$

$$(11c) \int_{E_0 - \Delta E}^{E_0} (E - E_T)^{1/2} dE - k \int_{E_c}^{E_0} (E - E_c)^{1/2} dE \quad . \quad E_c < E_0 < E_c + \Delta E$$

(11d,e,f) These are identical to (10d,e,f), respectively.

Integration of equations (10) and (11) give the following expressions:

Case I: $3N_c/2Z$ equals

$$(12a) \quad (E_0 - E_T)^{3/2} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad E_T < E_0 < E_c$$

$$(12b) \quad (E_0 - E_T)^{3/2} - k(E_0 - E_c)^{3/2} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad E_c < E_0 < E_T + \Delta E$$

$$(12c) \quad (E_0 - E_T)^{3/2} - (E_0 - E_T - \Delta E)^{3/2} - k(E_0 - E_c)^{3/2} \quad E_T + \Delta E < E_0 < E_c + \Delta E$$

$$(12d) \quad (E_0 - E_T)^{3/2} - (E_0 - E_T - \Delta E)^{3/2} - k \left[(E_0 - E_c)^{3/2} - (E_0 - E_c - \Delta E)^{3/2} \right] \\ . \quad . \quad . \quad . \quad . \quad . \quad E_c + \Delta E < E_0 < E_L$$

$$(12e) \quad 1/2 \left\{ (E_0 - E_T)^{3/2} - 2(E_0 - E_T - \Delta E)^{3/2} + (E_L - E_T)^{3/2} + b(E_0^{3/2} - E_L^{3/2}) \right.$$

$$\left. -k \left[(E_0 - E_c)^{3/2} - 2(E_0 - E_c - \Delta E)^{3/2} + (E_L - E_c)^{3/2} \right] \right\} E_L < E_0 < E_L + \Delta E$$

$$(12f) \quad 1/2 \left\{ (E_0 - E_T)^{3/2} - (E_0 - E_T - \Delta E)^{3/2} + b \left[E_0^{3/2} - (E_0 - \Delta E)^{3/2} \right] \right.$$

$$\left. -k \left[(E_0 - E_c)^{3/2} - (E_0 - E_c - \Delta E)^{3/2} \right] \right\} \quad . \quad . \quad . \quad E_0 > E_L + \Delta E$$

Case II: $3N_c/2Z$ equals

$$(13a) \quad (E_0 - E_T)^{3/2} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad E_T < E_0 < E + \Delta E$$

$$(13b) \quad (E_0 - E_T)^{3/2} - (E_0 - E_T - \Delta E)^{3/2} \quad . \quad . \quad . \quad E_T + \Delta E < E_0 < E_c$$

$$(13c) \quad (E_0 - E_T)^{3/2} - (E_0 - E_T - \Delta E)^{3/2} - k(E_0 - E_c)^{3/2} \quad E_c < E_0 < E_c + \Delta E$$

(13d,e,f) These are identical to (12d,e,f), respectively.

Case 1: $\beta_0 \neq 0$

$$(12a) \quad \beta_0 \neq 0 \quad \beta_0 > 0 \quad \beta_0 < 0$$

$$(12b) \quad \beta_0 \neq 0 \quad \beta_0 > 0 \quad \beta_0 < 0$$

$$(12c) \quad \beta_0 \neq 0 \quad \beta_0 > 0 \quad \beta_0 < 0$$

$$(12d) \quad \beta_0 \neq 0 \quad \beta_0 > 0 \quad \beta_0 < 0$$

$$\beta_0 > 0 \quad \beta_0 < 0$$

$$(12e) \quad \beta_0 \neq 0 \quad \beta_0 > 0 \quad \beta_0 < 0$$

$$\beta_0 > 0 \quad \beta_0 < 0$$

$$(12f) \quad \beta_0 \neq 0 \quad \beta_0 > 0 \quad \beta_0 < 0$$

$$\beta_0 > 0 \quad \beta_0 < 0$$

Case 2: $\beta_0 = 0$

$$(13a) \quad \beta_0 = 0 \quad \beta_0 > 0 \quad \beta_0 < 0$$

$$(13b) \quad \beta_0 = 0 \quad \beta_0 > 0 \quad \beta_0 < 0$$

$$(13c) \quad \beta_0 = 0 \quad \beta_0 > 0 \quad \beta_0 < 0$$

$$(13d) \quad \beta_0 = 0 \quad \beta_0 > 0 \quad \beta_0 < 0$$

These equations may be used for computing a theoretical yield curve. A word of caution must be injected at this point concerning equations (12e,f) and (13e,f). These have been derived from the assumption that $\sigma = C(E - E_T)^{1/2}$, and they will prove useful for reactions where that assumption is valid for the regions in question. The cross section for the $\text{Li}(p,n)$ reaction, however, is almost constant throughout portions of the regions to which these equations apply. This is no handicap, however, since the equations (12a-d) and (13a-d) enable one to compute a theoretical yield curve for target thicknesses up to 30 kev which is the limit of thickness being considered.

These equations may be used for computing a theoretical field curve. A word of caution must be injected at its origin concerning equations (12a, b) and (13a, b). These have been derived from the assumption that $\sigma = 0$ and $\mu = 1$. The assumption that $\sigma = 0$ is valid for the region of interest. The cross section for the $\mu = 1$ reaction, however, is almost constant throughout part one of the region to which these equations apply. This is in contrast, however, with the equations (12a, b) and (13a, b) which are to compute a theoretical field curve for the region of interest. It is for this reason that the equations are not considered.

VI. TARGET THICKNESS

A visual inspection of equations (12a,b) and (13a,b) shows that the geometric peak must occur for:

$$\text{Case I: } E_0 > E_T + \Delta E$$

$$\text{Case II: } E_0 > E_C$$

The requirement that the peak occur in the region defined by equations (12c) and (13c) for Cases I and II, respectively, is in either case:

$$N_c(\text{at } E_0 = E_C + \Delta E - \epsilon) > N_c(\text{at } E_0 = E_C + \Delta E)$$

where ϵ is an infinitesimal.

Utilizing equation (12c) or (13c), the requirement is:

$$(E_C - E_T + \Delta E - \epsilon)^{3/2} - (E_C - E_T - \epsilon)^{3/2} - k(\Delta E - \epsilon)^{3/2} >$$

$$(E_C - E_T + \Delta E)^{3/2} - (E_C - E_T)^{3/2} - k(\Delta E)^{3/2}$$

Expanding and collecting terms:

$$3/2 \quad \epsilon \left[(E_C - E_T)^{1/2} - (E_C - E_T + \Delta E)^{1/2} + k(\Delta E)^{1/2} \right] > 0$$

$$(E_C - E_T)^{1/2} + k(\Delta E)^{1/2} > (E_C - E_T + \Delta E)^{1/2}$$

Squaring both sides:

$$k^2 \Delta E + 2k(\Delta E)^{1/2}(E_C - E_T)^{1/2} > \Delta E$$

$$\left(\frac{1 - k^2}{2k}\right)^2 \Delta E < E_C - E_T$$

$$\Delta E < \frac{4(E_C - E_T)}{k^2 + 1/k^2 - 2}$$

Evaluation of the expression on the right-hand side of this inequality (Table IX, Appendix B) for values of θ between zero and 30 degrees, shows that this inequality holds for target thicknesses up to about 400 kev. Thus, the peak will occur in the regions defined by equations (12c) and (13c) for Cases I and II, respectively. Taking the derivative of N_C with respect to E_0 in this region and evaluating at the peak position:

$$\left. \frac{dN_C}{dE_0} \right|_{E_p} = 0, \text{ where } E_p \text{ is the incident particle energy at the}$$

peak position.

approximate value

$$K_{S,1} + 2K_{S,2} \sqrt{S_1} (n_1 - n_2) \sqrt{S_2} > 0$$

$$n_1 - n_2 > \frac{1}{2} \left(\frac{1}{S_1} - \frac{1}{S_2} \right)$$

$$\frac{(n_1 - n_2) \sqrt{S_1}}{S_1 + S_2} > 0$$

Verification of the expression in the right-hand side of (11) is made by using the inequality (Table 1, Appendix 2) for values of β between zero and 30 degrees, where the right-hand side of (11) is always positive. Thus, the inequality will be satisfied for all values of β up to about 30 degrees. If β is greater than 30 degrees, the inequality will be satisfied for values of β up to about 30 degrees. If β is greater than 30 degrees, the inequality will be satisfied for values of β up to about 30 degrees. Taking the derivative of the right-hand side of (11) with respect to β in the region of evaluation of the inequality

$$\frac{d}{d\beta} \left(\frac{(n_1 - n_2) \sqrt{S_1}}{S_1 + S_2} \right) > 0$$

we obtain

$$(14) \quad \Delta E = E_p - E_T - K$$

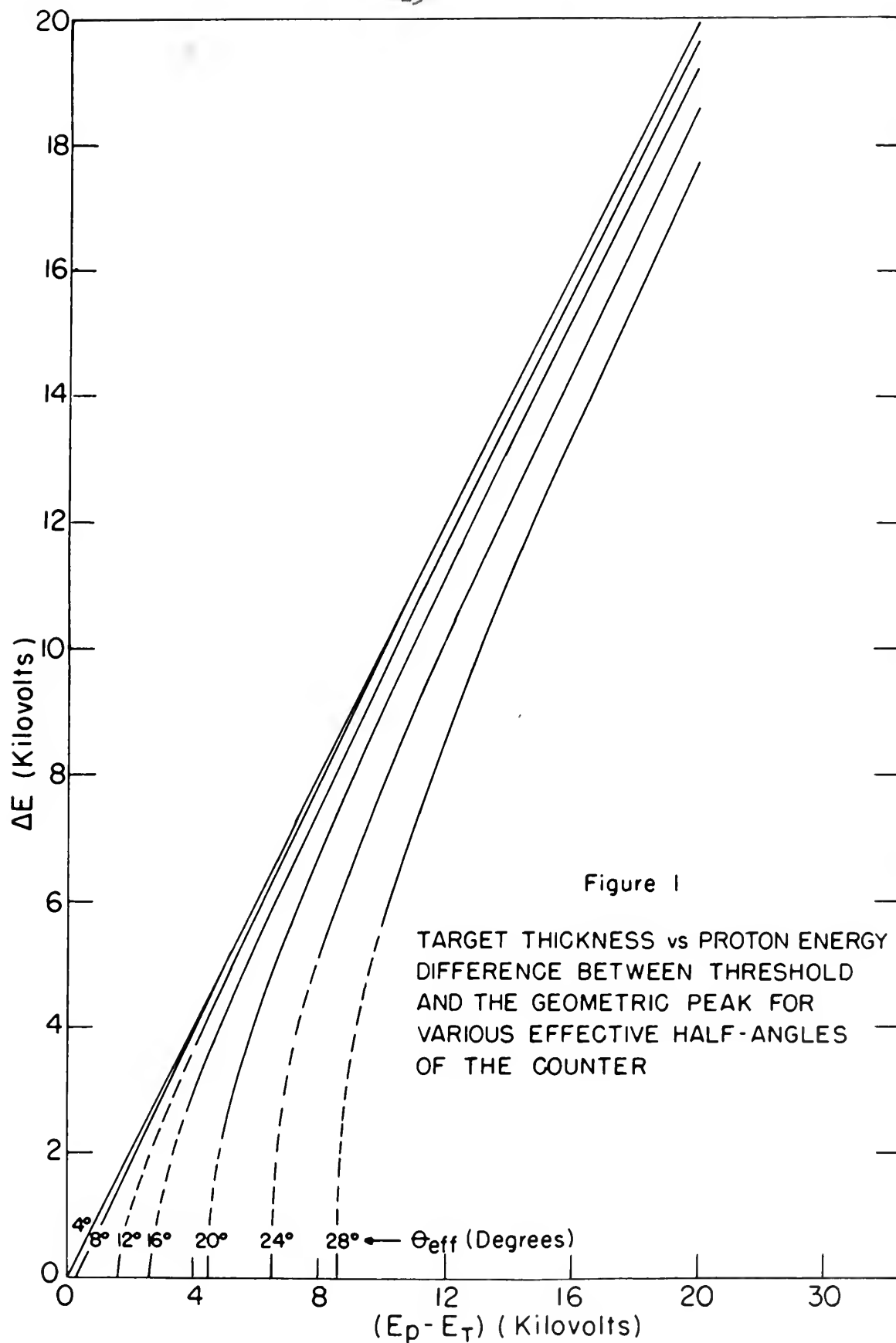
$$\begin{aligned} \text{where } K &= \left[(E_p - E_T)^{1/2} - k(E_p - E_c)^{1/2} \right]^2 \\ &= \left[(E_p - E_T)^{1/2} - \cos \theta E_T^{1/2} \left(\frac{E_p}{E_c} - 1 \right)^{1/2} \right]^2 \end{aligned}$$

The correction term (K) and the target thickness (ΔE) are evaluated in Table X for various values of the effective half-angle of the counter (θ_{eff}), and the proton energy difference between the geometric peak and the reaction threshold ($E_p - E_T$). The results are presented as a family of curves in Figure 1.

If one assumes that the cross section is given by:

$$(15) \quad \sigma = \text{const} \left(\frac{E - E_T}{E^5} \right)^{1/2}$$

Or, equally well, if one introduces the slowly varying constant $1/E^{5/2}$ under the integral sign of equations (10) and (11), then integration of these equations gives results that can be shown to be identical to the expressions given by Snowden and Whitehead² for Case I; they have made no mention of Case II. Proceeding as above, the derivative in the region of the peak leads to:



$$(16) \quad \Delta E \approx \frac{E_p - E_T - K}{1 - 5K/E_p} \approx E_p - E_T - K$$

in close agreement with equation (14). This is not surprising, since there is but little variation in the factor $(1/E^{5/2})$ over the relatively small range of energy being considered. Since these results add nothing new to the results previously obtained, the rather lengthy manipulations required for confirmation of the statements just made are omitted.

Equation (14) reduces to the result given by Hanson, Taschek, and Williams¹ when θ is small, namely:

$$(17) \quad \Delta E \approx E_p - E_T.$$

VII. PROTON ENERGY RESOLUTION

The result arrived at in equation (14) is based upon the assumption of an incident monergic proton beam with no proton straggling within the target. The effects will now be considered of an initial energy spread and straggling within the target.

The Rockefeller generator has an effective energy spread of about 0.075 percent with the entrance and exit slits set at 1.0 mm width⁸. Several microamperes of beam current are available with this energy definition. Better energy resolution may be obtained at the expense of beam current by using a narrower slit width.

In order to calculate the straggling it is necessary to know the stopping power (energy loss per unit weight per unit area). Bethe's^{9,10} treatment of energy loss, based upon the Born approximation, after correction for K shell binding, leads to the following equation in the nonrelativistic case:

$$(18) \quad -\frac{dE}{dX} = \frac{4\pi e^4 z^2 N}{mv^2} \left[Z \ln \left(\frac{2mv^2}{I} \right) - C_K \right]$$

where

- ze = charge of the incident particle
- v = velocity of the incident particle
- Z = the nuclear charge of the material
- m = electronic mass
- I = average excitation potential of the atom
- N = number of atoms per cm^3 of the material
- C_K = correction term to account for binding in K shell.

VII. ROTON ENERGY RELATION

The results arrived at in equation (11) is based upon the assumption of an incident wave packet of width δ and momentum p and a target of an initial energy spread and momentum width δ . The Roton energy relation has an effective energy spread of about 0.025 percent with the entrance and exit slit set at 1.0 mm width. Several experiments of beam current are available with this energy definition. Better energy resolution may be obtained at the expense of beam current by using a narrower slit width. In order to calculate the straggling it is necessary to know the stopping power, energy loss and energy spread. Bethe's^{9,10} treatment of energy loss, based upon the free electron theory, after correction for shell effects, has been the following equation in the nonrelativistic case:

$$(18) \quad \frac{dE}{dx} = \frac{4\pi e^4 N_A}{m_e^2 c^2} \left[\frac{Z}{A} \left(\ln \frac{2m_e c^2 \beta^2}{I} - \beta^2 \right) + \frac{1}{2} \beta^4 \right]$$

where

- N_A = number of atoms per gram
- Z = atomic number of the absorber
- A = atomic weight of the absorber
- I = the mean excitation potential of the absorber
- β = v/c where v is the velocity of the incident electron
- m_e = electron mass
- c = velocity of light
- e = electronic charge

This equation as it stands gives the energy loss per unit path length. This equation is used to obtain the stopping power of lithium relative to beryllium for which the absolute value of stopping power is well known¹⁰. This immediately gives:

$$(19) \quad \frac{-\frac{dE}{d(\rho X)}_{Li}}{-\frac{dE}{d(\rho X)}_{Be}} = \frac{W_o \left[Z \ln \left(\frac{2mv^2}{I} \right) - C_K \right]}{W \left[Z_o \ln \left(\frac{2mv^2}{I_o} \right) - C_{K_o} \right]} = 1.09$$

where ρ = density, W = atomic weight, and the subscript "o" denotes the standard which in this case is beryllium. The calculation follows:

$$\text{Relative stopping power} = \frac{B/W}{B_o/W_o}$$

where B and B_o are the quantities within the brackets of numerator and denominator, respectively.

$$2mv^2 = \left(\frac{4m}{M} \right) \left(\frac{Mv^2}{2} \right) = \frac{E_p}{459}$$

where M and E_p are the mass and energy of the proton, respectively.

$$\text{Taking } E_p = 1900 \text{ kev} = 1.9 \times 10^6 \text{ ev,}$$

$$\ln \left(\frac{E_p}{459} \right) = \ln 4139 = 8.324$$

This equation as it stands gives the stopping power as a function of path length. This equation is used to obtain the stopping power of lithium relative to paraffin for which the atomic number of paraffin is known. This is written as follows:

$$(19) \quad \frac{\left[\frac{dE}{dx} \right]_{\text{Li}}}{\left[\frac{dE}{dx} \right]_{\text{paraffin}}} = \frac{\left[\frac{dE}{dx} \right]_{\text{paraffin}}}{\left[\frac{dE}{dx} \right]_{\text{paraffin}}} \quad \text{where } \frac{dE}{dx} \text{ is the stopping power}$$

where $\frac{dE}{dx}$ is the stopping power, Z is the atomic number, and A is the atomic weight. The equation follows:

$$\frac{dE}{dx} = \frac{1}{A} \left(\frac{Z}{A} \right)^2 \left(\frac{dE}{dx} \right)_{\text{paraffin}}$$

where Z and A are the atomic number and atomic weight of the element, and $\left(\frac{dE}{dx} \right)_{\text{paraffin}}$ is the stopping power of paraffin.

$$\left(\frac{dE}{dx} \right)_{\text{Li}} = \left(\frac{Z}{A} \right)^2 \left(\frac{dE}{dx} \right)_{\text{paraffin}}$$

where $\left(\frac{dE}{dx} \right)_{\text{Li}}$ is the stopping power of lithium, and $\left(\frac{dE}{dx} \right)_{\text{paraffin}}$ is the stopping power of paraffin.

$$\left(\frac{dE}{dx} \right)_{\text{Li}} = \left(\frac{Z}{A} \right)^2 \left(\frac{dE}{dx} \right)_{\text{paraffin}}$$

$$\left(\frac{dE}{dx} \right)_{\text{Li}} = \left(\frac{Z}{A} \right)^2 \left(\frac{dE}{dx} \right)_{\text{paraffin}}$$

$$\ell_n I = \ell_n 34 = 3.526$$

$$\ell_n I_0 = \ell_n 60.4 = 4.101$$

The above I values are from Table 4, reference (9).

$$C_k \simeq C_{k_0} = 0.405 \quad (\text{Table II-1, reference (10)}).$$

Therefore,

$$\frac{B}{B_0} = \frac{14.394 - 0.405}{16.892 - 0.405} = 0.8485.$$

$$\text{Rel. Stop. Pwr (Li to Be)} = \frac{W_0}{W} (0.8485) = \frac{9.015}{7.018} (0.8485) = 1.09$$

Using the value 144 for the absolute stopping power of beryllium for 1900-kev protons (Table III-7, reference (10)), one obtains for lithium:

$$\text{Stopping Power (Li)} = 144 \times 1.09 = 157 \frac{\text{kev cm}^2}{\text{mg}}$$

An initially homogeneous beam of protons, after passing through a target of thickness t will have an energy distribution with a standard deviation Ω given by¹¹,

$$g_{11} = g_{22} = 1$$

$$g_{12} = g_{21} = 0$$

The above metric is a flat metric.

$$g_{11} = g_{22} = 1, g_{12} = g_{21} = 0$$

Therefore,

$$\frac{1}{g} = \frac{1}{1 \cdot 1 - 0 \cdot 0} = 1$$

$$\text{Vol. form} = \sqrt{g} dx^1 \wedge dx^2 = dx^1 \wedge dx^2$$

Let us consider the volume element $dx^1 \wedge dx^2$ in the x^1, x^2 plane. The volume element is a parallelogram with sides dx^1 and dx^2 . The area of this parallelogram is $dx^1 dx^2$.

Now, let us consider the volume element $dx^1 \wedge dx^2$ in the x^1, x^2 plane. The volume element is a parallelogram with sides dx^1 and dx^2 . The area of this parallelogram is $dx^1 dx^2$. This is the same as the volume element in the x^1, x^2 plane.

$$(20) \quad \Omega^2 = \frac{4\pi e^4}{M} \left(\frac{Z}{A}\right) t$$

where M and e are the mass and charge of the proton, Z and A are the atomic and mass numbers of the target element, respectively. The thickness t is in weight per unit area.

$$(21) \quad \Omega = \sqrt{\frac{2\pi e^4}{M}} \left(\frac{2Z}{A}\right)^{1/2} t^{1/2} = 8.85 \left(\frac{2Z}{A}\right)^{1/2} t^{1/2}$$

where $\frac{2Z}{A} \approx 1$ for all elements except hydrogen.

$$\frac{2Z}{A} \text{ (lithium)} = \left(\frac{6}{7}\right)^{1/2} = 0.93.$$

(22) $\Omega = 8.23 t^{1/2}$ for lithium; where Ω is given in kev, and t is in mg/cm^2 .

Since the stopping power of lithium was calculated to be

$$157 \frac{\text{kev cm}^2}{\text{mg}}$$

we may write:

(23) $\Omega = 0.657 (\Delta E)^{1/2}$ where Ω is in kev and ΔE is the target thickness in kev. From this equation, the following table is computed:

$$\Omega = \frac{1}{2} \left(\frac{1}{\lambda} + \frac{1}{\lambda'} \right) \quad (20)$$

where λ and λ' are the wave lengths in the incident and reflected beams, respectively. The difference δ in the wave lengths is given by

$$\delta = \frac{1}{2} \left(\frac{1}{\lambda} - \frac{1}{\lambda'} \right) \quad (21)$$

where δ is the difference in the wave lengths.

$$\delta = \frac{1}{2} \left(\frac{1}{\lambda} - \frac{1}{\lambda'} \right) \quad (22)$$

$$\delta = \frac{1}{2} \left(\frac{1}{\lambda} - \frac{1}{\lambda'} \right) \quad (23)$$

is the difference in the wave lengths.

From the above equations it can be seen that

$$\frac{\delta}{\lambda} = \frac{1}{2} \left(\frac{1}{\lambda} - \frac{1}{\lambda'} \right) \quad (24)$$

we have

$$\delta = \frac{1}{2} \left(\frac{1}{\lambda} - \frac{1}{\lambda'} \right) \quad (25)$$

this is the difference in the wave lengths.

hence

$\Delta E(\text{kev})$	(kev)	$2 \Gamma_s(\text{kev})$
20	2.94	6.94
15	2.54	5.98
10	2.08	4.90
5	1.47	3.46
4	1.31	3.09
3	1.14	2.69
2	0.93	2.19
1	0.66	1.55
0.5	0.46	1.08

Assuming a normal distribution, the half-width at half maximum (Γ_s) is given by:

$$\Gamma_s = 1.178 \Omega = 0.774 (\Delta E)^{1/2}.$$

The value of $2 \Gamma_s$ is given in the above table for ready comparison with the proton energy spread of the beam, which is about 1.4 kev, for an 0.075 percent energy definition.

It is now desired to determine the minimum value of target thickness for which equation (14) may be expected to be valid when threshold is to be determined by extrapolating the linear portion of the yield curve to the axis. It is unlikely that a precise theoretical treatment can be given. However, an approximate value for this

γ	$\gamma(\gamma)$	$\gamma(\gamma)$
0.0	0.0	0.0
0.1	0.1	0.1
0.2	0.2	0.2
0.3	0.3	0.3
0.4	0.4	0.4
0.5	0.5	0.5
0.6	0.6	0.6
0.7	0.7	0.7
0.8	0.8	0.8
0.9	0.9	0.9
1.0	1.0	1.0

... ..

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$$\gamma = \dots$$

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minimum thickness may be obtained by qualitative arguments based upon some rather crude assumptions, in the following manner.

The geometric peak is treated as a resonance peak with full width at half maximum of the order of $1.4 (E_p - E_T)$. It is shown (Section 3D of reference (9)) that for a resolution equal to or less than the width of half maximum of the resonance peak, the main effect is to depress the peak without changing the slope of the linear portion of the curve. Applying this condition to the geometric peak for a resolution of 2Γ , it follows that for

$$\Gamma < 0.7 \Delta E < 0.7(E_p - E_T)$$

extrapolation to the axis of the linear portion of the rise curve should still give a good value of the reaction threshold. Although the peak has been depressed, there will be no appreciable shift in peak position, inasmuch as the geometric peak is only slightly asymmetrical. Thus, the value of $(E_p - E_T)$ obtained by extrapolation is essentially the same value as would have been obtained with no resolution, and equation (14) will be valid.

The resolution (2Γ) used in the foregoing discussion is the resultant of energy spread in the beam and straggling in the target and will be of the order of the larger of these two resolutions, considered separately. A slightly better value is obtained if it is assumed that these two resolutions add as Gaussians:

minimum thickness may be obtained by substituting an assumed value for

some relation of the parameters, in the following manner.

The geometric peak is treated as a resonance peak with full

width of half maximum of the order of 1.1 ($\Delta p = 0.1$). It is assumed

(Section 2) of reference (2) that for a resolution equal to 1.1

than the width of half maximum of the resonance peak, the width of the

is to decrease the peak without changing the area of the line.

tion of the curve. According to the condition for the geometric peak for

a resolution of 1.1, it follows that for

$$1.1 < \Delta p < 0.1$$

extrapolation to the side of the theory pointing to the line

should still use a good value of the width of the peak.

the peak is not resolved, there will be no resolution with in

peak position, therefore, in the present case it will be

approximately. The value of Δp (constant) is determined

then is essentially the same value as would have been obtained with

no resolution, and section (1) will be valid.

A second case (2) is that in which the resolution is

the resolution of the peak is in the range of 1.1 to 1.5

target, and still in the range of 1.1 to 1.5

theoretical, compared to the experimental.

It is to be noted that the value of Δp is not the same as

$$\Gamma^2 = \Gamma_s^2 + \Gamma_i^2$$

where Γ_i is the half-width at half maximum of the energy distribution in the beam. Actually, the shape of the energy distribution in the beam is largely dependent upon the exit slit width of the magnetic analyzer (private communication with W. M. Preston), departing from an approximate Gaussian as the slit width is decreased.

Applying the foregoing results to the specific case of a 2-kev target thickness and a 1-mm slit width gives:

$$2\Gamma = [(2.19)^2 + (1.4)^2]^{1/2} = 2.6 \text{ kev.}$$

Since $\Gamma = 1.3$ kev, the condition for validity of equation (14)

$$\Gamma < 0.7 \Delta E \quad \text{becomes}$$

$$\Gamma < 1.4 \text{ kev,}$$

and this inequality holds for the conditions stated.

The conclusion to be drawn from the foregoing is that equation (14) will be valid for target thicknesses of about 2 kev minimum, when the threshold is determined by extrapolation and slit widths of 1 mm are used on the Rockefeller generator.

For $\Gamma > 0.7 (E_p - E_T)$, extrapolation to the axis gives an apparent threshold energy E_T' which is less than E_T since the slope of the curve is decreased. Assuming that the peak position is not

$$L_1 = L_2 = L_3$$

where L is the length of half-wave of the medium. The
 in the beam. The shape of the wave is determined by
 the beam is nearly constant when the width of the wave
 netic analyzer. The wave is constant when L is constant, and
 from an approximate relation on the width of the wave.

Applying the theory to the wave in the case of a
 large thickness and low wave number:

$$L = \frac{1}{2} \left[\frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} - 1 \right] \quad (1)$$

Since L is constant, and v is constant, (1)

$$L < \frac{1}{2} \left[\frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} - 1 \right]$$

$$L < \frac{1}{2} \left[\frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} - 1 \right]$$

and the wave number is constant, and the wave number is constant.

The wave number is constant, and the wave number is constant.

(2) With a constant wave number, the wave number is constant.

The wave number is constant, and the wave number is constant.

are the same for the wave number.

It is $L < \frac{1}{2} \left[\frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} - 1 \right]$

Applying the theory to the wave in the case of a

of the wave number is constant, and the wave number is constant.

shifted because of resolution, equation (14) becomes:

$$(25) \quad \Delta E = E_p - E_T - K = E_p - E_T' - K - \delta E_T$$

where δE_T is the difference between the true and apparent threshold. This could be determined experimentally for various values of resolution and target thickness. It is apparent, however, that equation (14) may still be used with reasonable results if the position of E_T is accurately known. An accurate method for determining the absolute value of E_T is to measure the yield over the "rise" portion of the geometric peak of a target which is several times thicker than the proton energy resolution of the beam ($\Delta E \sim 20$ kev), using an effective half-angle of the counter, such that $E_0 - E_T \sim 4$ kev. Equation (13a) then applies over the lower portion of the curve where the shape is not affected by the relatively small resolution.

$$N_c \propto (E_0 - E_T)^{3/2}$$

A plot of $N_c^{2/3}$ should thus be linear and its extrapolation to the axis should give an accurate value of E_T . The extreme lower portion of the yield curve must of course be neglected as it has been distorted by the incident proton energy spread.

shifted because of resolution, equation (1) becomes

$$\delta^2 = \frac{1}{2} \left(\frac{1}{R_1} + \frac{1}{R_2} - \frac{1}{R_3} \right) \quad (22)$$

where δ is the difference between the two apparent thicknesses. This could be determined exactly using the values of R_1 and R_2 then and target thickness. It is assumed, however, that R_1 may still be used with reasonable accuracy in the calculation of δ . An accurate method for determining the absolute value of δ is to assume $R_1 = R_2$ and the position of the geometric mean of R_1 and R_2 is found. Then R_1 and R_2 are proton energy measurements. The values of R_1 and R_2 are then half-angle of the emission angle $\theta = 0$ and $\theta = 180^\circ$ respectively (Fig. 1) and applied to the values of R_1 and R_2 the same as the values of R_1 and R_2 are used.

$$\delta = \frac{1}{2} \left(\frac{1}{R_1} + \frac{1}{R_2} - \frac{1}{R_3} \right)$$

A value of δ is obtained from the expression above. This value should be subtracted from the values of R_1 and R_2 of the two apparent thicknesses. The values of R_1 and R_2 are then used to find the values of R_1 and R_2 and the values of R_1 and R_2 are used to find the values of R_1 and R_2 .

VIII. THE CALIBRATED LONG COUNTER

The use of a calibrated long counter for determining the thickness of lithium targets is mentioned by Hanson, Taschek, and Williams¹. Aging of the target caused by oxidation, contamination, and the like causes the geometric peak to shift toward higher proton energies, since the total number of atoms has increased. However, the yield at proton energies well above threshold is essentially the same for either the fresh or the aged target, since the number of lithium atoms has not changed while the geometric effects tend to disappear with increasing proton energy. This is attributed to the fact that at proton energies greater than E_L , the neutrons are emitted more nearly isotropically in the laboratory, and the distribution of neutron energies is more nearly uniform over the face of the counter. The change in the shape of the yield curve as a target ages is clearly shown in Figure 17 of reference (1).

It is important to note that target thickness as obtained by a calibrated counter may be used for determining the number of target atoms for either a fresh or aged target. However, the target thickness for purposes of determining the neutron resolution (arising from this target thickness) can be obtained with a calibrated counter only for a fresh target. Conversely, the "rise" method gives target thickness for purposes of determining the neutron resolution (arising from target thickness) for both fresh or aged targets; whereas it leads

to a determination of the number of target (lithium) atoms only for fresh targets. Hence, the necessity of using fresh targets is apparent when one desires to calibrate a counter in terms of counting rate per unit target thickness, if thickness is determined by the rise method for the purpose of calibration.

A measurement of the thickness of a fresh target by the "rise" method followed by a measurement of the yield at a proton energy above 1930 kev in a region where the cross section is nearly constant serves to calibrate the counter in terms of counting rate per unit target thickness. One may select a region of nearly constant cross section either above or below the 2.24-Mev resonance. The region above the peak should give better results as the geometric effect is less pronounced than in the region below the peak.

A calculation will be made to determine the linearity of neutron yield with respect to target thickness for various counter positions for an incident proton energy (E_0) of 1960 kev. This particular value is selected not only because the cross section will be nearly constant for all possible E values within targets of thickness 0 to 30 kev, but also because equation (8f) will apply throughout since $E > E_L$ for all possible E values within the target.

$$(8f) \quad N_0 = Z \int_{E_0 - \Delta E}^{E_0} G_3 \sigma \, dE \dots \dots \dots E > E_L$$

to be determined by the nature of the work to be done and the character of the material to be handled. The work should be so organized that the worker can do it with the least possible fatigue and in the shortest possible time. The work should be so organized that the worker can do it with the least possible fatigue and in the shortest possible time. The work should be so organized that the worker can do it with the least possible fatigue and in the shortest possible time.

1. The purpose of the study was to determine the effect of the treatment on the growth of the plants.

1. The first of these is the fact that the
2. second of these is the fact that the
3. third of these is the fact that the
4. fourth of these is the fact that the
5. fifth of these is the fact that the
6. sixth of these is the fact that the
7. seventh of these is the fact that the
8. eighth of these is the fact that the
9. ninth of these is the fact that the
10. tenth of these is the fact that the

(24)

11

and since σ is nearly constant:

$$(26) \quad N_C = Z \sigma \int_{E_0 - \Delta E}^{E_0} G_3 dE.$$

As the proton energy increases, G_3 approaches a limit which is constant for a given half-angle of the counter. Thus, the counting rate (N_C) approaches $(\sigma Z G_3) \Delta E = \text{const } \Delta E$.

If, however, we evaluate N_C at $E_0 = 1960$ kev,

$$(27) \quad N_C = \frac{\sigma Z}{2} \int_{E_0 - \Delta E}^{E_0} \left[1 - \cos \theta \left\{ 1 - \frac{m_1 m_3}{m_2 m_4} \sin^2 \theta \left(\frac{E}{E - E_T} \right) \right\}^{1/2} \right. \\ \left. + \sin^2 \theta \left(\frac{m_1 m_3}{m_2 m_4} \right)^{1/2} \left(\frac{E}{E - E_T} \right)^{1/2} \right] dE$$

Another form of the expression for G_3 is used here (equation 36c of Appendix A):

$$\frac{m_1 m_3}{m_2 m_4} \sim 0.02$$

$$(\sin \theta)_{\max}^2 \sim 0.25$$

$$\frac{E}{E - E_T} \sim 25$$

Therefore:

and since α is nearly constant:

$$(26) \quad \int_{\alpha_0}^{\alpha} \frac{1}{\alpha^2} d\alpha = \frac{1}{\alpha_0} - \frac{1}{\alpha}$$

As the proper energy increases, α increases a little which is constant for a given value of α_0 . Thus, the constant rate (26) approaches $\frac{1}{\alpha_0}$ as α goes to infinity. If, however, we consider α of the order of α_0 , we

$$(27) \quad \int_{\alpha_0}^{\alpha} \frac{1}{\alpha^2} d\alpha = \frac{1}{\alpha_0} - \frac{1}{\alpha} = \frac{1}{\alpha_0} \left(1 - \frac{\alpha_0}{\alpha} \right)$$

$$= \frac{1}{\alpha_0} \left(1 - \frac{\alpha_0}{\alpha} \right) \approx \frac{1}{\alpha_0} \left(1 - \frac{\alpha_0}{\alpha} \right)$$

where the first term of the expansion is $\frac{1}{\alpha_0}$ and the second term is $-\frac{\alpha_0}{\alpha^2}$.

$$\frac{1}{\alpha_0} \approx \frac{1}{\alpha} \left(1 + \frac{\alpha_0}{\alpha} \right) \quad \text{or} \quad \alpha \approx \alpha_0 \left(1 + \frac{\alpha_0}{\alpha} \right)$$

Therefore:

$$\left[\frac{m_1 m_3}{m_2 m_4} \sin^2 \theta \left(\frac{E}{E - E_T} \right) \right]_{\max} \sim 0.125; \text{ hence, only two terms are}$$

needed in the series expansion of the brackets containing this factor:

$$(28) \quad N_c = \frac{\sigma Z}{2} \left[(1 - \cos \theta) \int_{E_0 - \Delta E}^{E_0} dE + \frac{\cos \theta \sin^2 \theta}{2} \left(\frac{m_1 m_3}{m_2 m_4} \right) \int_{E_0 - \Delta E}^{E_0} \frac{E}{E - E_T} dE \right. \\ \left. + \sin^2 \theta \left(\frac{m_1 m_3}{m_2 m_4} \right)^{1/2} \int_{E_0 - \Delta E}^{E_0} \left(\frac{E}{E - E_T} \right)^{1/2} dE \right]$$

First integral:

$$\int_{E_0 - \Delta E}^{E_0} dE = \Delta E$$

Second integral:

$$\int_{E_0 - \Delta E}^{E_0} \frac{E}{E - E_T} dE = \left[E + E_T \ln(E - E_T) \right]_{E_0 - \Delta E}^{E_0} = \Delta E + E_T \ln \left[\frac{E_0 - E_T}{E_0 - E_T - \Delta E} \right]$$

$$\lim_{\epsilon \rightarrow 0} \left[\frac{1}{\epsilon} \left(\frac{1}{\epsilon} \int_{-\epsilon}^{\epsilon} f(x) dx - f(0) \right) \right] = f'(0)$$

where $f(x)$ is a function of x and $f'(0)$ is the derivative of $f(x)$ at $x=0$.

$$\lim_{\epsilon \rightarrow 0} \left[\frac{1}{\epsilon} \left(\frac{1}{\epsilon} \int_{-\epsilon}^{\epsilon} f(x) dx - f(0) \right) \right] = f'(0) \quad (55)$$

$$\lim_{\epsilon \rightarrow 0} \left[\frac{1}{\epsilon} \left(\frac{1}{\epsilon} \int_{-\epsilon}^{\epsilon} f(x) dx - f(0) \right) \right] = f'(0)$$

where $f(x)$ is a function of x and $f'(0)$ is the derivative of $f(x)$ at $x=0$.

$$\lim_{\epsilon \rightarrow 0} \left[\frac{1}{\epsilon} \left(\frac{1}{\epsilon} \int_{-\epsilon}^{\epsilon} f(x) dx - f(0) \right) \right] = f'(0)$$

where $f(x)$ is a function of x and $f'(0)$ is the derivative of $f(x)$ at $x=0$.

$$\lim_{\epsilon \rightarrow 0} \left[\frac{1}{\epsilon} \left(\frac{1}{\epsilon} \int_{-\epsilon}^{\epsilon} f(x) dx - f(0) \right) \right] = f'(0)$$

Third integral:

$$\int_{E_0 - \Delta E}^{E_0} \left(\frac{E}{E - E_T} \right)^{1/2} dE = E_0^{1/2} \int \frac{dE}{(E - E_T)^{1/2}} \quad \text{since } E^{1/2} \text{ is}$$

nearly constant over the range of integration. This is readily integrated:

$$E_0^{1/2} \int_{E_0 - \Delta E}^{E_0} \frac{dE}{(E - E_T)^{1/2}} = 2E_0^{1/2} (E - E_T)^{1/2} \Bigg|_{E_0 - \Delta E}^{E_0}$$

$$= 2E_0^{1/2} \left[(E_0 - E_T)^{1/2} - (E_0 - E_T - \Delta E)^{1/2} \right]$$

Neglecting all terms except the first two in the expansion of

$\left[(E_0 - E_T) - \Delta E \right]^{1/2}$, one obtains:

$$\int_{E_0 - \Delta E}^{E_0} \frac{E^{1/2}}{(E - E_T)^{1/2}} dE \approx \frac{\Delta E}{(1 - E_T/E_0)^{1/2}}$$

Third integral:

$$\int_{-\infty}^{\infty} \frac{1}{(x^2 + a^2)^2} dx = \frac{\pi}{2a^3}$$

which can be obtained by the method of residues.

integrated:

$$\int_{-\infty}^{\infty} \frac{1}{(x^2 + a^2)^2} dx = \frac{\pi}{2a^3}$$

$$= \frac{\pi}{2a^3}$$

which can be obtained by the method of residues.

$$\left[\frac{1}{(x^2 + a^2)^2} \right]_{-\infty}^{\infty} = 0$$

$$\int_{-\infty}^{\infty} \frac{1}{(x^2 + a^2)^2} dx = \frac{\pi}{2a^3}$$

$$\begin{aligned}
 (29) \quad N_c = \frac{\sigma \cdot Z}{2} \left\{ \left[1 - \cos \theta + \frac{\cos \theta \sin^2 \theta}{2} \left(\frac{m_1 m_3}{m_2 m_L} \right) \right. \right. \\
 \left. \left. + \left(1 - \frac{E_T}{E_0} \right)^{-1/2} \sin^2 \theta \left(\frac{m_1 m_3}{m_2 m_L} \right)^{1/2} \right] \Delta E \right. \\
 \left. + \frac{\cos \theta \sin^2 \theta}{2} \left(\frac{m_1 m_3}{m_2 m_L} \right) E_T \ln \left[\frac{E_0 - E_T}{E_0 - E_T - \Delta E} \right] \right\}
 \end{aligned}$$

$$\begin{aligned}
 (30) \quad N_c = \frac{\sigma \cdot Z}{2} \left\{ \left[1 - \cos \theta + 0.01032 \cos \theta \sin^2 \theta + 0.71850 \sin^2 \theta \right] \Delta E \right. \\
 \left. + 19.385 \ln \left(\frac{78}{78 - \Delta E} \right) \cos \theta \sin^2 \theta \right\}
 \end{aligned}$$

Equation (30) is evaluated for various θ and various ΔE in Tables VI through VIII, the final results being displayed in Table VIII. From this table, it is seen that the counting rate per unit target thickness is practically constant for any given half-angle of counter. The results expressed in Table VIII are plotted to give the curve of Figure 2. The linearity of counting rate with respect to target thickness for a given half-angle of the counter is displayed in Tables VII (A-H).

$$\left\{ \frac{\left(\frac{M_{12}}{M_{11}} \right) \frac{0.5 \pi \times 2 \times 800}{5} + 0.001 - \right\} \frac{0.5 \pi}{5} = 0.16 \quad (25)$$

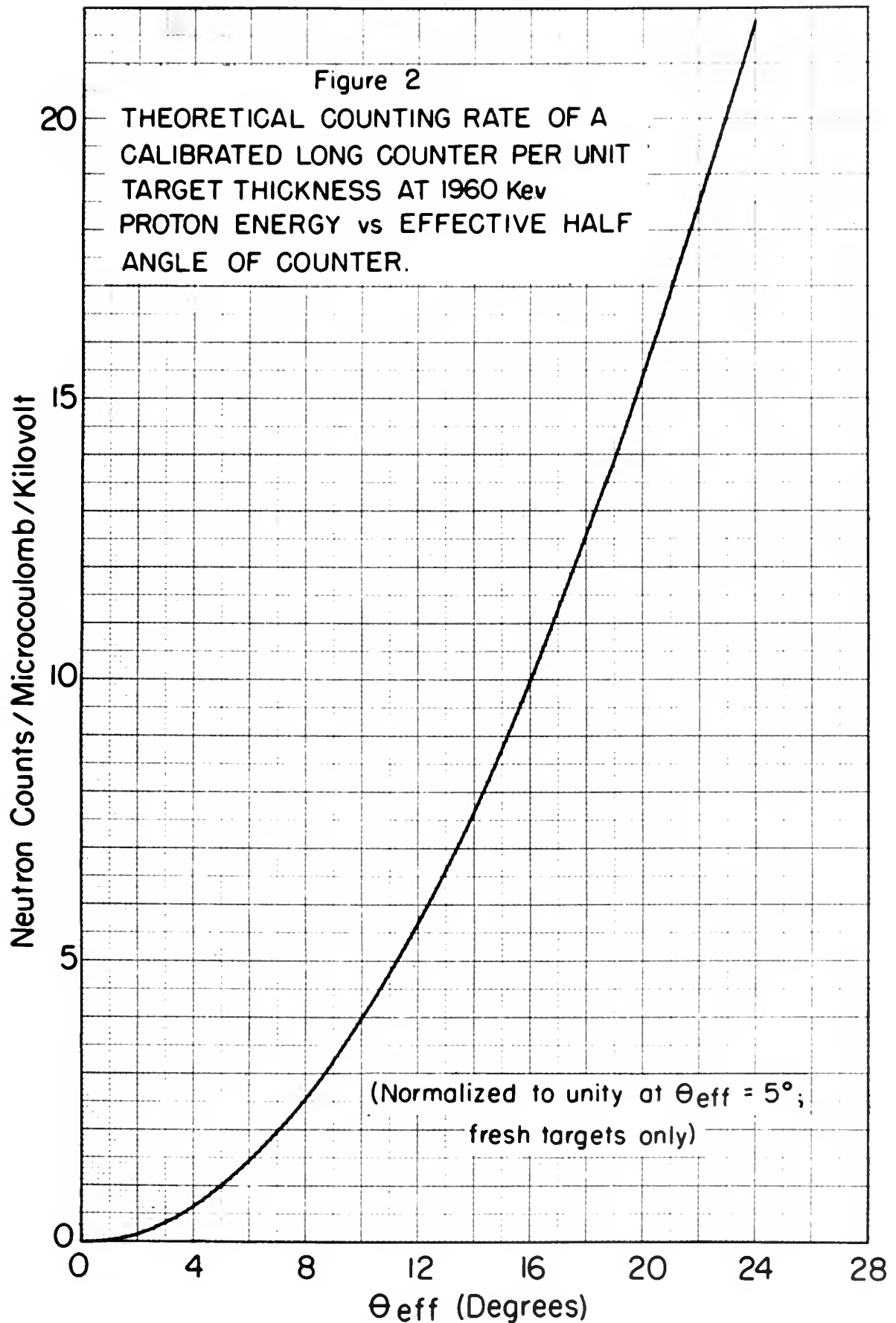
$$= \left[\frac{5}{2} (2.17) - 2.17 \right] - \left(\frac{2}{2} - 1 \right) +$$

$$\left\{ \left[\frac{1}{1 - \frac{1}{2}} \right] \right\} = 2$$

[illegible]

1. The first part of the document is a list of names and titles, including "The Hon. Mr. Justice" and "The Hon. Mr. Justice".

1. The first part of the document is a letter from the President of the United States to the Congress, dated January 1, 1861. It is a very important document, as it contains the President's message to the Congress at the beginning of his first term. The letter is written in a formal, dignified style, and it is one of the most important documents in American history.



IX. THE EXPERIMENT AND RESULTS

The purpose of the experiment was:

1. To determine the effective half-angle of the long counter used in this laboratory (7.5-inch paraffin diameter completely shielded with cadmium to reduce background of thermal neutrons) as a function of the distance from the target. With this counter there may be a slight dependence on target thickness; it was desired to check this point.

2. To apply equation (14) to experimental yield curves of targets of several known thicknesses for various counter positions in order to check the validity of this equation.

A convenient method of determining the effective half-angle of the counter for various positions is as follows: Adjust the values of target thickness and counter half-angle to obtain a best fit between the theoretical yield curve³ (equations 12 and 13 apply) and the experimental curve for one relatively large half-angle of the counter. A calibrated counter placed successively at various distances from the target will give a set of relative counting rates per unit target thickness. These relative counting rates, in conjunction with Figure 2, determine a family of possible curves of effective half-angle versus distance from the target. The correct curve is selected from this family by using the value of θ_{eff} found in fitting the theoretical and experimental curves, as mentioned above.

IX. THE DISCRETE CASE

The name of the argument was:

1. To determine the discrete part of the integral

counter used in the integral (1.1) is equal to the value of the function of the distance from the origin. This value is equal to the value of the function of the distance from the origin. This value is equal to the value of the function of the distance from the origin.

2. To determine the discrete part of the integral

of the function of the distance from the origin. This value is equal to the value of the function of the distance from the origin. This value is equal to the value of the function of the distance from the origin.

in order to determine the discrete part of the integral. This value is equal to the value of the function of the distance from the origin. This value is equal to the value of the function of the distance from the origin.

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from the function of the distance from the origin. This value is equal to the value of the function of the distance from the origin. This value is equal to the value of the function of the distance from the origin.

Three lithium targets of various thicknesses were prepared by evaporating the metal in vacuum onto a tantalum backing rotating eccentrically with the beam axis. This apparatus is mounted on the beam exit tube of the Rockefeller generator, making it unnecessary to expose the target to the atmosphere at any time. The need for fresh targets in correlating target thickness by the "rise" method and by calibrated counter is discussed in Section VIII.

An experimental yield curve was determined for each target and for each of the following distances from counter to the target: 39.4, 18, 11, and 7.5 inches. (The 11- and 18-inch yield curves were omitted on the third target because of lack of time.)

The frequency meter normally used for selecting the desired proton energy was inoperative on the day allotted to this experiment. The standby frequency meter, which was used, had an indeterminable frequency drift that caused the proton energy to be in error by a varying amount up to 3 kev maximum. Because of this, the yield curves from two of the three targets were completely unreliable from the viewpoint of this experiment. The remaining target gave reasonably smooth curves, but they are not considered to be accurate enough for determining the effective half-angle by fitting a theoretical curve. The yields obtained are tabulated in Tables I (A-D) and the experimental curves are presented as Figures 3, 4, 5, and 6. In the regions of interest, enough counts were taken so that the standard deviation could be neglected.

A regulated high-voltage supply was the source of the 2250 volts applied to the BF_3 detector. (This voltage corresponds to the center of the plateau for that particular counter.) The counting rate with a Ra-Be standard source on top of the counter was determined at intervals over a period of three days and never varied more than 2.6 percent.

The BF_3 detector was a Model 3 hOE Mark 2, manufactured by Radiation Counter Laboratories. The output pulses were fed through a preamplifier to a Model 100 amplifier, thence to a Model 1060 multiscaler manufactured by the Atomic Instrument Company. The discriminator is an integral part of the complete multiscaler unit.

Counts were taken for an integral number of microcoulombs of charge on the target. If the beam current changed appreciably during a run, the count was repeated.

Background was small enough to be neglected throughout this experiment.

In an attempt to derive from this experiment some information that would be of immediate use to this laboratory, it was decided to proceed with the assumption that equation (14) is valid and to select that particular curve from the family of possible curves of effective half-angle versus distance from the target which would produce the observed shift in peak position at the various counter positions.

Since the yield is slow varying in the vicinity of 1960-kev proton energy, the counting rates are hardly affected by the frequency drift experienced and may be used for all three targets. In Table VIII, it is shown that the counting rate per unit target thickness is independent of target thickness for the counter half-angles used. An average value of counting rate per unit target thickness for the three targets is calculated, and the relative counting rate determined.

TABLE I (A)

Experimental Data
Counter at 00, 39.4 inches from Target

<u>Target</u>	<u>Frequency (Megacycles)</u>	<u>Counts</u>	<u>μCoulombs</u>
No. 2	10.468	48	40
	.470	507	40
	.471	2438	40
	.473	5566	40
	.476	7487	40
	.479	8923	40
	.482	10004	40
	.485	10967	40
	.487	22508	80
	.489	22428	80
	.491	21181	80
	.495	19523	80
	.500	8349	40
	.680	9382	160
No. 1	10.680	6431	80
No. 3	10.680	24297	80

(A) I. 112.7

Horizontal distance of 20.11 inches from base

Vertical distance	Horizontal distance	Vertical distance	Horizontal distance
0.0	0.0	0.0	0.0
0.1	0.1	0.1	0.1
0.2	0.2	0.2	0.2
0.3	0.3	0.3	0.3
0.4	0.4	0.4	0.4
0.5	0.5	0.5	0.5
0.6	0.6	0.6	0.6
0.7	0.7	0.7	0.7
0.8	0.8	0.8	0.8
0.9	0.9	0.9	0.9
1.0	1.0	1.0	1.0
1.1	1.1	1.1	1.1
1.2	1.2	1.2	1.2
1.3	1.3	1.3	1.3
1.4	1.4	1.4	1.4
1.5	1.5	1.5	1.5
1.6	1.6	1.6	1.6
1.7	1.7	1.7	1.7
1.8	1.8	1.8	1.8
1.9	1.9	1.9	1.9
2.0	2.0	2.0	2.0
2.1	2.1	2.1	2.1
2.2	2.2	2.2	2.2
2.3	2.3	2.3	2.3
2.4	2.4	2.4	2.4
2.5	2.5	2.5	2.5
2.6	2.6	2.6	2.6
2.7	2.7	2.7	2.7
2.8	2.8	2.8	2.8
2.9	2.9	2.9	2.9
3.0	3.0	3.0	3.0
3.1	3.1	3.1	3.1
3.2	3.2	3.2	3.2
3.3	3.3	3.3	3.3
3.4	3.4	3.4	3.4
3.5	3.5	3.5	3.5
3.6	3.6	3.6	3.6
3.7	3.7	3.7	3.7
3.8	3.8	3.8	3.8
3.9	3.9	3.9	3.9
4.0	4.0	4.0	4.0
4.1	4.1	4.1	4.1
4.2	4.2	4.2	4.2
4.3	4.3	4.3	4.3
4.4	4.4	4.4	4.4
4.5	4.5	4.5	4.5
4.6	4.6	4.6	4.6
4.7	4.7	4.7	4.7
4.8	4.8	4.8	4.8
4.9	4.9	4.9	4.9
5.0	5.0	5.0	5.0
5.1	5.1	5.1	5.1
5.2	5.2	5.2	5.2
5.3	5.3	5.3	5.3
5.4	5.4	5.4	5.4
5.5	5.5	5.5	5.5
5.6	5.6	5.6	5.6
5.7	5.7	5.7	5.7
5.8	5.8	5.8	5.8
5.9	5.9	5.9	5.9
6.0	6.0	6.0	6.0
6.1	6.1	6.1	6.1
6.2	6.2	6.2	6.2
6.3	6.3	6.3	6.3
6.4	6.4	6.4	6.4
6.5	6.5	6.5	6.5
6.6	6.6	6.6	6.6
6.7	6.7	6.7	6.7
6.8	6.8	6.8	6.8
6.9	6.9	6.9	6.9
7.0	7.0	7.0	7.0
7.1	7.1	7.1	7.1
7.2	7.2	7.2	7.2
7.3	7.3	7.3	7.3
7.4	7.4	7.4	7.4
7.5	7.5	7.5	7.5
7.6	7.6	7.6	7.6
7.7	7.7	7.7	7.7
7.8	7.8	7.8	7.8
7.9	7.9	7.9	7.9
8.0	8.0	8.0	8.0
8.1	8.1	8.1	8.1
8.2	8.2	8.2	8.2
8.3	8.3	8.3	8.3
8.4	8.4	8.4	8.4
8.5	8.5	8.5	8.5
8.6	8.6	8.6	8.6
8.7	8.7	8.7	8.7
8.8	8.8	8.8	8.8
8.9	8.9	8.9	8.9
9.0	9.0	9.0	9.0
9.1	9.1	9.1	9.1
9.2	9.2	9.2	9.2
9.3	9.3	9.3	9.3
9.4	9.4	9.4	9.4
9.5	9.5	9.5	9.5
9.6	9.6	9.6	9.6
9.7	9.7	9.7	9.7
9.8	9.8	9.8	9.8
9.9	9.9	9.9	9.9
10.0	10.0	10.0	10.0

TABLE I(B)

Experimental Data
Counter at 0°, 18 inches from Target

<u>Target</u>	<u>Frequency (Megacycles)</u>	<u>Counts</u>	<u>μCoulombs</u>
No. 2	10.478	206	40
	.481	4017	40
	.482	9937	40
	.484	20669	40
	.486	26345	40
	.488	33213	40
	.490	39450	40
	.492	41834	40
	.494	43471	40
	.496	44300	40
	.498	45990	40
	.500	45287	40
	.502	44162	40
	.510	34790	40
	.680	17534	80
No. 1	10.680	25320	80
No. 3	10.680	-	-

TABLE I (C)

Experimental Data
Counter at 0°, 11 inches from Target

<u>Target</u>	<u>Frequency (Megacycles)</u>	<u>Counts</u>	<u>μCoulombs</u>
No. 2	10.478	586	20
	.480	3864	20
	.483	14506	20
	.485	22813	20
	.489	36572	20
	.493	44621	20
	.496	48686	20
	.498	53171	20
	.500	52869	20
	.502	52078	20
	.506	48899	20
	.680	40968	80
No. 1	10.680	64856	80
No. 3	10.680	-	-

TABLE I (C)

Number of 0's, 1's and 2's in the sequence

Number of 0's	Number of 1's	Number of 2's	Sequence
0	0	0	000
0	1	0	001
0	2	0	010
0	3	0	011
0	4	0	100
0	5	0	101
0	6	0	110
0	7	0	111
0	8	0	200
0	9	0	201
0	10	0	210
0	11	0	211
0	12	0	300
0	13	0	301
0	14	0	310
0	15	0	311
0	16	0	400
0	17	0	401
0	18	0	410
0	19	0	411
0	20	0	500
0	21	0	501
0	22	0	510
0	23	0	511
0	24	0	600
0	25	0	601
0	26	0	610
0	27	0	611
0	28	0	700
0	29	0	701
0	30	0	710
0	31	0	711
0	32	0	800
0	33	0	801
0	34	0	810
0	35	0	811
0	36	0	900
0	37	0	901
0	38	0	910
0	39	0	911
0	40	0	1000
0	41	0	1001
0	42	0	1010
0	43	0	1011
0	44	0	1100
0	45	0	1101
0	46	0	1110
0	47	0	1111
0	48	0	1200
0	49	0	1201
0	50	0	1210
0	51	0	1211
0	52	0	1300
0	53	0	1301
0	54	0	1310
0	55	0	1311
0	56	0	1400
0	57	0	1401
0	58	0	1410
0	59	0	1411
0	60	0	1500
0	61	0	1501
0	62	0	1510
0	63	0	1511
0	64	0	1600
0	65	0	1601
0	66	0	1610
0	67	0	1611
0	68	0	1700
0	69	0	1701
0	70	0	1710
0	71	0	1711
0	72	0	1800
0	73	0	1801
0	74	0	1810
0	75	0	1811
0	76	0	1900
0	77	0	1901
0	78	0	1910
0	79	0	1911
0	80	0	2000
0	81	0	2001
0	82	0	2010
0	83	0	2011
0	84	0	2100
0	85	0	2101
0	86	0	2110
0	87	0	2111
0	88	0	2200
0	89	0	2201
0	90	0	2210
0	91	0	2211
0	92	0	2300
0	93	0	2301
0	94	0	2310
0	95	0	2311
0	96	0	2400
0	97	0	2401
0	98	0	2410
0	99	0	2411
0	100	0	2500
0	101	0	2501
0	102	0	2510
0	103	0	2511
0	104	0	2600
0	105	0	2601
0	106	0	2610
0	107	0	2611
0	108	0	2700
0	109	0	2701
0	110	0	2710
0	111	0	2711
0	112	0	2800
0	113	0	2801
0	114	0	2810
0	115	0	2811
0	116	0	2900
0	117	0	2901
0	118	0	2910
0	119	0	2911
0	120	0	3000
0	121	0	3001
0	122	0	3010
0	123	0	3011
0	124	0	3100
0	125	0	3101
0	126	0	3110
0	127	0	3111
0	128	0	3200
0	129	0	3201
0	130	0	3210
0	131	0	3211
0	132	0	3300
0	133	0	3301
0	134	0	3310
0	135	0	3311
0	136	0	3400
0	137	0	3401
0	138	0	3410
0	139	0	3411
0	140	0	3500
0	141	0	3501
0	142	0	3510
0	143	0	3511
0	144	0	3600
0	145	0	3601
0	146	0	3610
0	147	0	3611
0	148	0	3700
0	149	0	3701
0	150	0	3710
0	151	0	3711
0	152	0	3800
0	153	0	3801
0	154	0	3810
0	155	0	3811
0	156	0	3900
0	157	0	3901
0	158	0	3910
0	159	0	3911
0	160	0	4000
0	161	0	4001
0	162	0	4010
0	163	0	4011
0	164	0	4100
0	165	0	4101
0	166	0	4110
0	167	0	4111
0	168	0	4200
0	169	0	4201
0	170	0	4210
0	171	0	4211
0	172	0	4300
0	173	0	4301
0	174	0	4310
0	175	0	4311
0	176	0	4400
0	177	0	4401
0	178	0	4410
0	179	0	4411
0	180	0	4500
0	181	0	4501
0	182	0	4510
0	183	0	4511
0	184	0	4600
0	185	0	4601
0	186	0	4610
0	187	0	4611
0	188	0	4700
0	189	0	4701
0	190	0	4710
0	191	0	4711
0	192	0	4800
0	193	0	4801
0	194	0	4810
0	195	0	4811
0	196	0	4900
0	197	0	4901
0	198	0	4910
0	199	0	4911
0	200	0	5000

TABLE I (D)

Experimental Data
Counter at 0°, 7.5 inches from Target

<u>Target</u>	<u>Frequency (Megacycles)</u>	<u>Counts</u>	<u>μCoulombs</u>
No. 2	10.4855	2644	20
	.488	14370	20
	.491	29869	20
	.494	47787	20
	.497	62822	20
	.499	68224	20
	.501	72367	20
	.503	81286	20
	.505	86557	20
	.507	85522	20
	.509	84755	20
	.511	82615	20
	.515	73769	20
	.680	33912	40
No. 1	10.680	49807	40
No. 3	10.680	43949	10

TABLE I (D)

Center of 00, 1.5 inches from forest
 ground level

Time	Temperature (°C)	Humidity (%)	Wind Speed (m/s)
10.0	10.0	10.0	10.0
10.5	10.5	10.5	10.5
11.0	11.0	11.0	11.0
11.5	11.5	11.5	11.5
12.0	12.0	12.0	12.0
12.5	12.5	12.5	12.5
13.0	13.0	13.0	13.0
13.5	13.5	13.5	13.5
14.0	14.0	14.0	14.0
14.5	14.5	14.5	14.5
15.0	15.0	15.0	15.0
15.5	15.5	15.5	15.5
16.0	16.0	16.0	16.0
16.5	16.5	16.5	16.5
17.0	17.0	17.0	17.0
17.5	17.5	17.5	17.5
18.0	18.0	18.0	18.0
18.5	18.5	18.5	18.5
19.0	19.0	19.0	19.0
19.5	19.5	19.5	19.5
20.0	20.0	20.0	20.0
20.5	20.5	20.5	20.5
21.0	21.0	21.0	21.0
21.5	21.5	21.5	21.5
22.0	22.0	22.0	22.0
22.5	22.5	22.5	22.5
23.0	23.0	23.0	23.0
23.5	23.5	23.5	23.5
24.0	24.0	24.0	24.0
24.5	24.5	24.5	24.5
25.0	25.0	25.0	25.0
25.5	25.5	25.5	25.5
26.0	26.0	26.0	26.0
26.5	26.5	26.5	26.5
27.0	27.0	27.0	27.0
27.5	27.5	27.5	27.5
28.0	28.0	28.0	28.0
28.5	28.5	28.5	28.5
29.0	29.0	29.0	29.0
29.5	29.5	29.5	29.5
30.0	30.0	30.0	30.0
30.5	30.5	30.5	30.5
31.0	31.0	31.0	31.0
31.5	31.5	31.5	31.5
32.0	32.0	32.0	32.0
32.5	32.5	32.5	32.5
33.0	33.0	33.0	33.0
33.5	33.5	33.5	33.5
34.0	34.0	34.0	34.0
34.5	34.5	34.5	34.5
35.0	35.0	35.0	35.0
35.5	35.5	35.5	35.5
36.0	36.0	36.0	36.0
36.5	36.5	36.5	36.5
37.0	37.0	37.0	37.0
37.5	37.5	37.5	37.5
38.0	38.0	38.0	38.0
38.5	38.5	38.5	38.5
39.0	39.0	39.0	39.0
39.5	39.5	39.5	39.5
40.0	40.0	40.0	40.0
40.5	40.5	40.5	40.5
41.0	41.0	41.0	41.0
41.5	41.5	41.5	41.5
42.0	42.0	42.0	42.0
42.5	42.5	42.5	42.5
43.0	43.0	43.0	43.0
43.5	43.5	43.5	43.5
44.0	44.0	44.0	44.0
44.5	44.5	44.5	44.5
45.0	45.0	45.0	45.0
45.5	45.5	45.5	45.5
46.0	46.0	46.0	46.0
46.5	46.5	46.5	46.5
47.0	47.0	47.0	47.0
47.5	47.5	47.5	47.5
48.0	48.0	48.0	48.0
48.5	48.5	48.5	48.5
49.0	49.0	49.0	49.0
49.5	49.5	49.5	49.5
50.0	50.0	50.0	50.0
50.5	50.5	50.5	50.5
51.0	51.0	51.0	51.0
51.5	51.5	51.5	51.5
52.0	52.0	52.0	52.0
52.5	52.5	52.5	52.5
53.0	53.0	53.0	53.0
53.5	53.5	53.5	53.5
54.0	54.0	54.0	54.0
54.5	54.5	54.5	54.5
55.0	55.0	55.0	55.0
55.5	55.5	55.5	55.5
56.0	56.0	56.0	56.0
56.5	56.5	56.5	56.5
57.0	57.0	57.0	57.0
57.5	57.5	57.5	57.5
58.0	58.0	58.0	58.0
58.5	58.5	58.5	58.5
59.0	59.0	59.0	59.0
59.5	59.5	59.5	59.5
60.0	60.0	60.0	60.0
60.5	60.5	60.5	60.5
61.0	61.0	61.0	61.0
61.5	61.5	61.5	61.5
62.0	62.0	62.0	62.0
62.5	62.5	62.5	62.5
63.0	63.0	63.0	63.0
63.5	63.5	63.5	63.5
64.0	64.0	64.0	64.0
64.5	64.5	64.5	64.5
65.0	65.0	65.0	65.0
65.5	65.5	65.5	65.5
66.0	66.0	66.0	66.0
66.5	66.5	66.5	66.5
67.0	67.0	67.0	67.0
67.5	67.5	67.5	67.5
68.0	68.0	68.0	68.0
68.5	68.5	68.5	68.5
69.0	69.0	69.0	69.0
69.5	69.5	69.5	69.5
70.0	70.0	70.0	70.0
70.5	70.5	70.5	70.5
71.0	71.0	71.0	71.0
71.5	71.5	71.5	71.5
72.0	72.0	72.0	72.0
72.5	72.5	72.5	72.5
73.0	73.0	73.0	73.0
73.5	73.5	73.5	73.5
74.0	74.0	74.0	74.0
74.5	74.5	74.5	74.5
75.0	75.0	75.0	75.0
75.5	75.5	75.5	75.5
76.0	76.0	76.0	76.0
76.5	76.5	76.5	76.5
77.0	77.0	77.0	77.0
77.5	77.5	77.5	77.5
78.0	78.0	78.0	78.0
78.5	78.5	78.5	78.5
79.0	79.0	79.0	79.0
79.5	79.5	79.5	79.5
80.0	80.0	80.0	80.0
80.5	80.5	80.5	80.5
81.0	81.0	81.0	81.0
81.5	81.5	81.5	81.5
82.0	82.0	82.0	82.0
82.5	82.5	82.5	82.5
83.0	83.0	83.0	83.0
83.5	83.5	83.5	83.5
84.0	84.0	84.0	84.0
84.5	84.5	84.5	84.5
85.0	85.0	85.0	85.0
85.5	85.5	85.5	85.5
86.0	86.0	86.0	86.0
86.5	86.5	86.5	86.5
87.0	87.0	87.0	87.0
87.5	87.5	87.5	87.5
88.0	88.0	88.0	88.0
88.5	88.5	88.5	88.5
89.0	89.0	89.0	89.0
89.5	89.5	89.5	89.5
90.0	90.0	90.0	90.0
90.5	90.5	90.5	90.5
91.0	91.0	91.0	91.0
91.5	91.5	91.5	91.5
92.0	92.0	92.0	92.0
92.5	92.5	92.5	92.5
93.0	93.0	93.0	93.0
93.5	93.5	93.5	93.5
94.0	94.0	94.0	94.0
94.5	94.5	94.5	94.5
95.0	95.0	95.0	95.0
95.5	95.5	95.5	95.5
96.0	96.0	96.0	96.0
96.5	96.5	96.5	96.5
97.0	97.0	97.0	97.0
97.5	97.5	97.5	97.5
98.0	98.0	98.0	98.0
98.5	98.5	98.5	98.5
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99.5	99.5	99.5	99.5
100.0	100.0	100.0	100.0

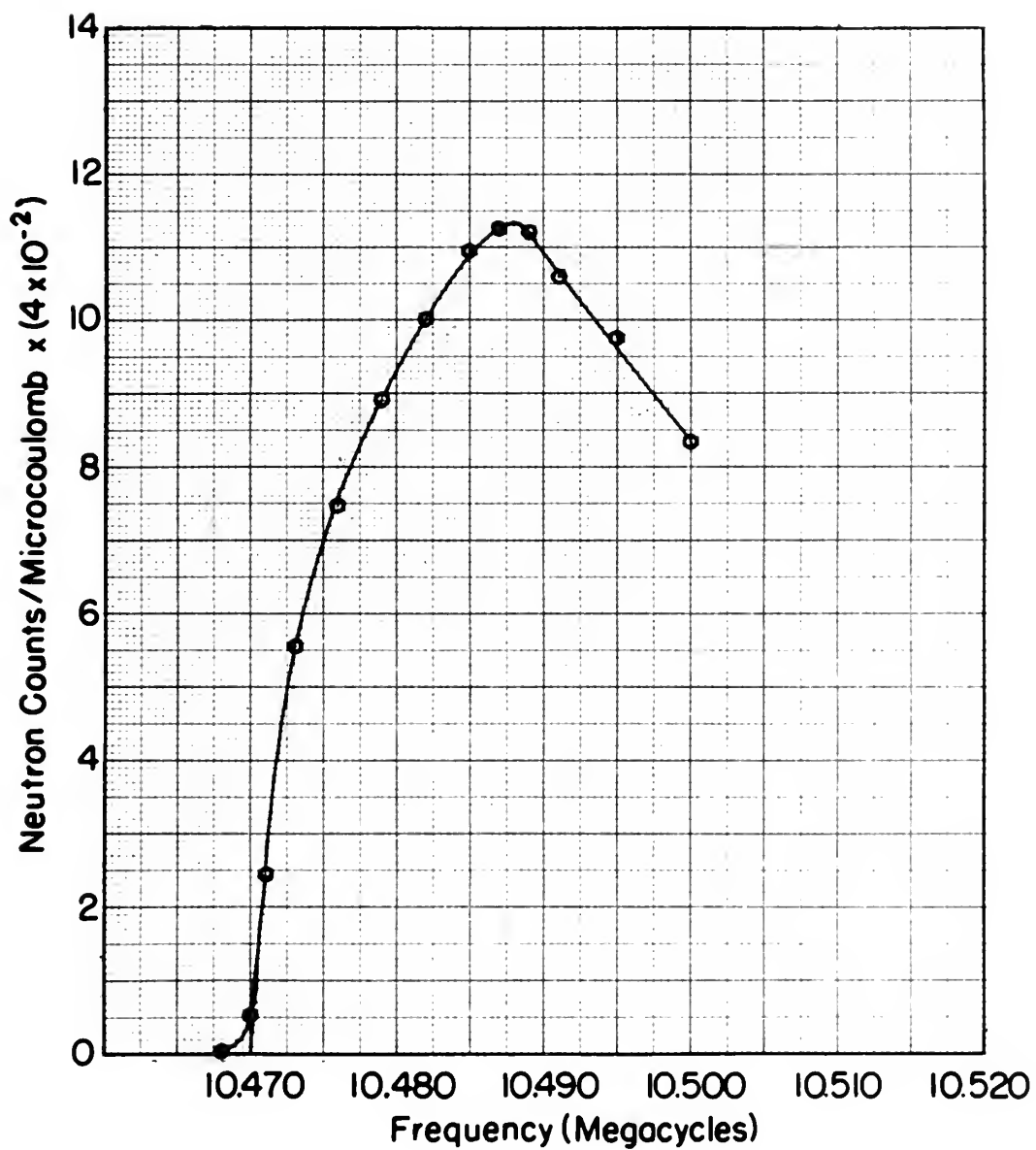


Figure 3

Neutron Yield from $\text{Li}^7 (p,n) \text{Be}^7$

Target Thickness 6.44 Kev

Distance from Target to Counter = 39.4 inches



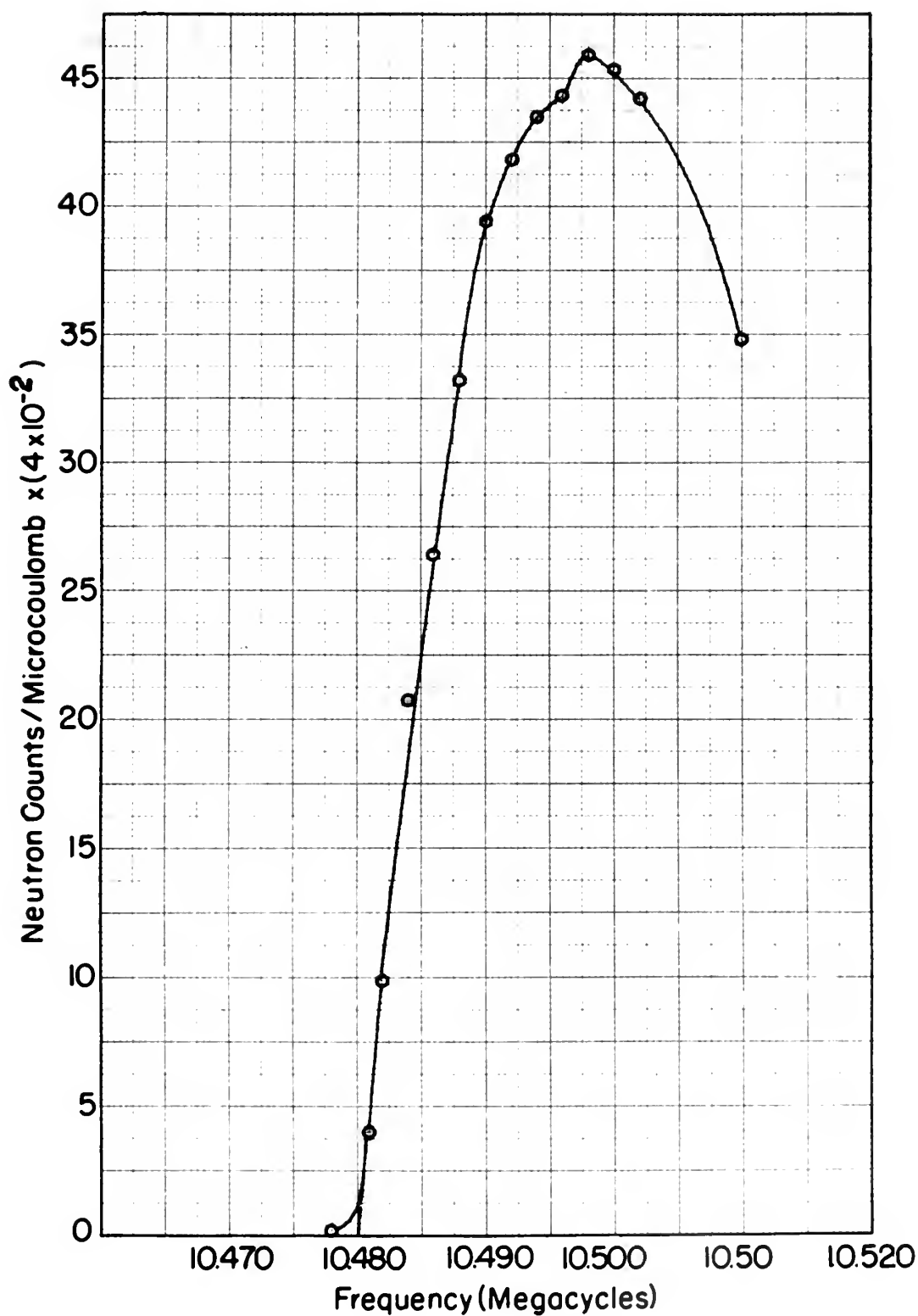


Figure 4

Neutron Yield from $\text{Li}^7(p,n)\text{Be}^7$

Target Thickness 6.44 Kev

Distance from Target to Counter = 18.0 inches

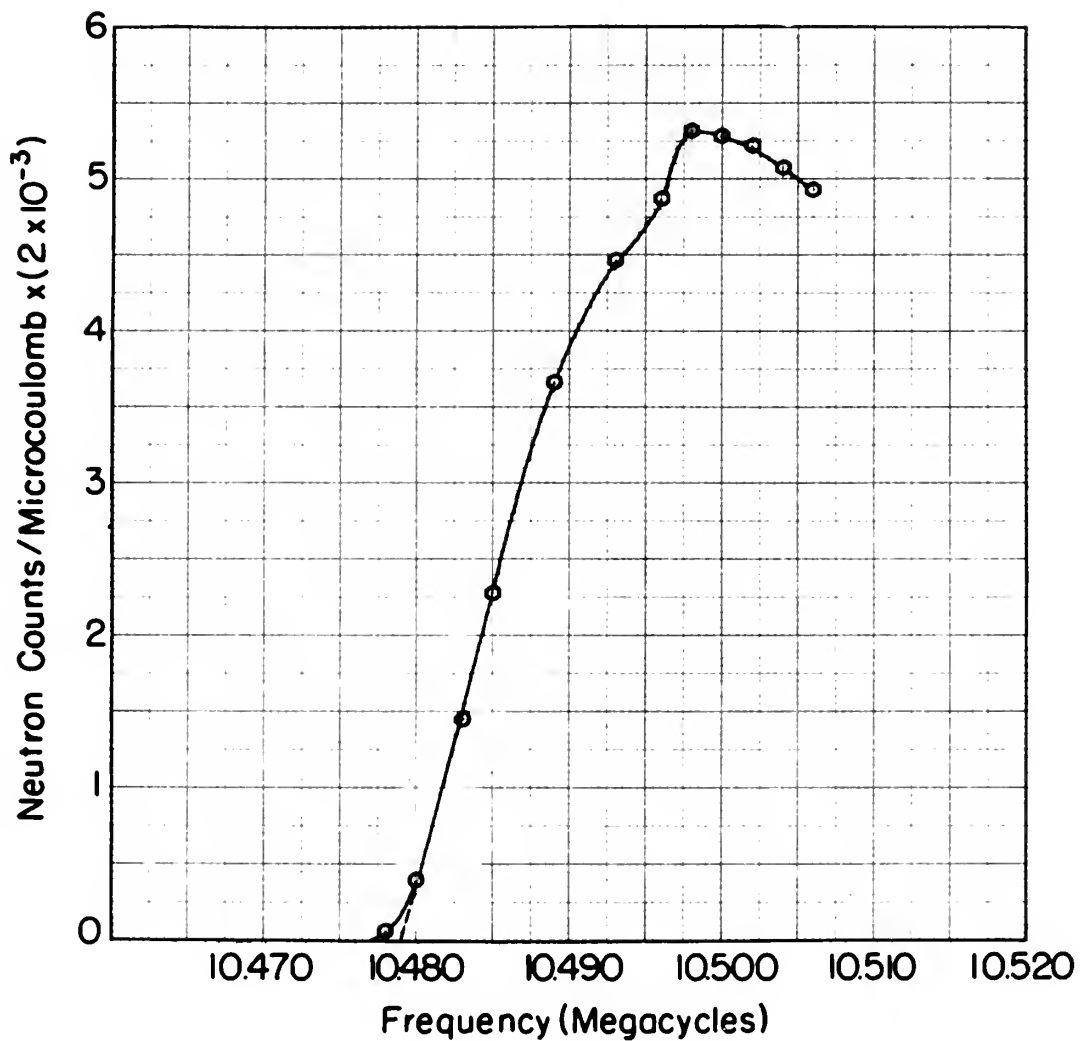


Figure 5

Neutron Yield from $\text{Li}^7(p,n)\text{Be}^7$

Target Thickness 6.44 Kev

Distance from Target to Counter = 11.0 inches



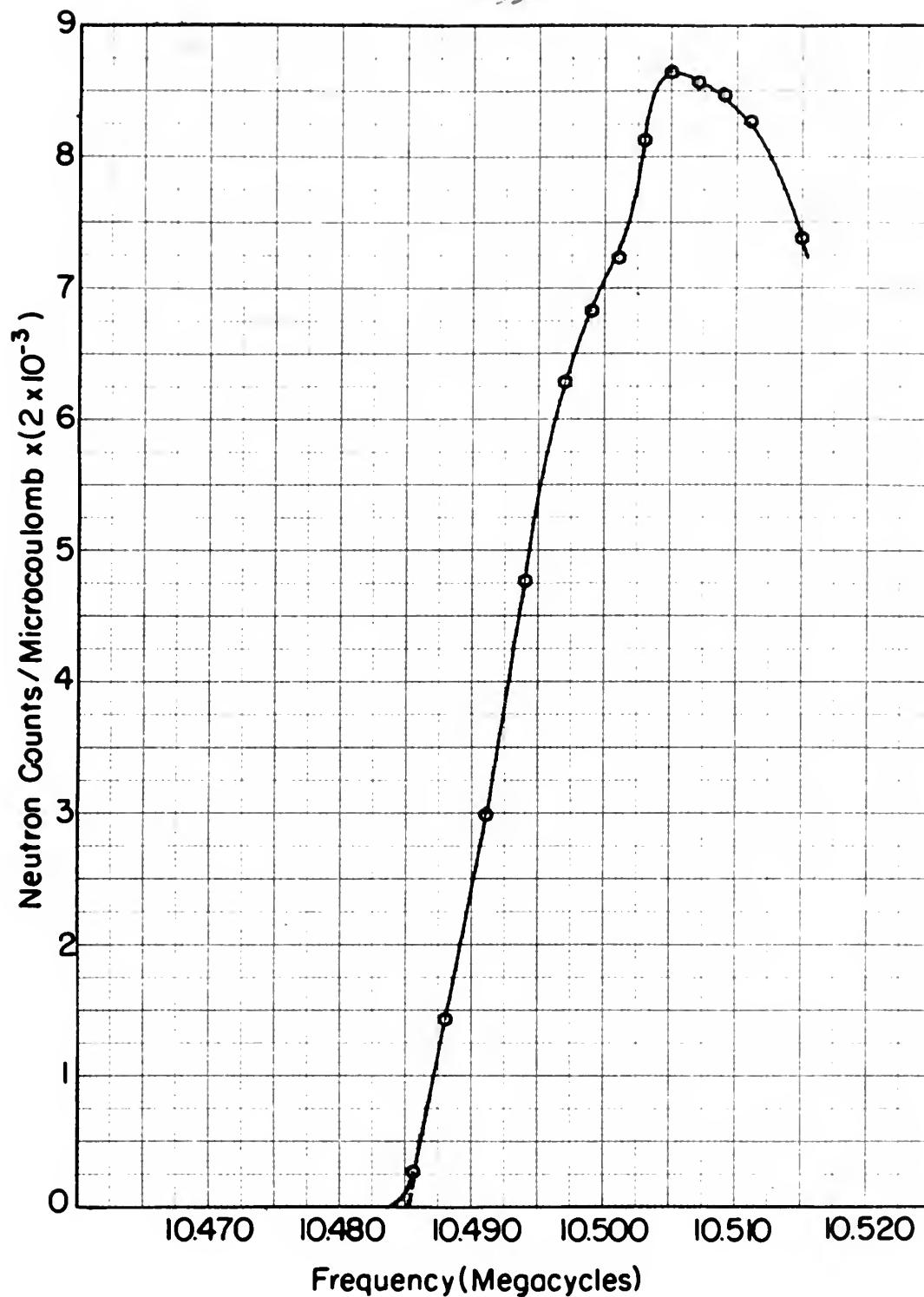


Figure 6

Neutron Yield from $\text{Li}^7(p,n)\text{Be}^7$

Target Thickness 6.44 Kev

Distance from Target to Counter = 7.5 inches

Distance from Target	Counts per 80 μ Coulombs at 1953-kev Proton Energy		
	Target 1	Target 2	Target 3
39.4"	6431	4691	24297
18.0	25320	17534	-
11.0	64856	40968	-
7.5	99614	67824	351592

Since the counting rate per unit target thickness should be constant at any given distance from the target, the number unity is arbitrarily assigned to the values obtaining at 39.4 inches and the ratios calculated with reference to it.

Distance from Target	Relative Counting Rates per Unit Target Thickness			
	Target 1	Target 2	Target 3	Average
39.4"	1	1	1	1
18.0	3.94	3.74	-	3.84
11.0	10.08	8.73	-	9.41
7.5	15.49	14.46	14.47	14.81

These average relative counting rates are maintained but are normalized to fit the curve of Figure 2 so that the effective half-angle has in succession the values 3° , 4° , and 5° at 39.4 inches from the target.

Distance from Target	Target 1	Target 2	Target 3
39.0	61.31	66.01	65.57
18.0	52.350	57.31	-
11.0	61.856	60.55	-
7.5	60.71	67.61	35.75

Since the counting rate per unit target thickness should be constant at any given distance from the target, the number unity is arbitrarily assumed as the value obtained at 39.0 inches and the ratios calculated with reference to it.

Distance from Target	Target 1	Target 2	Target 3	Average
39.0	1	1	1	1
18.0	0.85	0.85	0.85	0.85
11.0	0.85	0.85	0.85	0.85
7.5	0.85	0.85	0.85	0.85

These ratios relative to unity are the ratios used for the purpose of the above table. The ratios are calculated with reference to the value obtained at 39.0 inches and the ratios calculated with reference to it.

Counter Distance	Aver. Ratios	Ratio	$\theta^{\circ}_{\text{eff}}$	Ratio	$\theta^{\circ}_{\text{eff}}$	Ratio	$\theta^{\circ}_{\text{eff}}$
39.4"	1	.361	3.0	.63	4.0	1	5
18.0	3.84	1.386	5.8	2.42	7.7	3.84	9.8
11.0	9.41	3.397	9.2	5.93	12.2	9.41	15.5
7.5	14.81	5.346	11.6	9.33	15.4	14.81	19.6

From these values are drawn the three curves of the family of curves of effective half-angle versus distance from the target (Figure 7).

The difference between the peak and threshold positions in kilocycles is taken directly from the yield curves. These frequency differences (ΔF) are converted into proton energy differences by the relation:

$$(31) \quad (E_p - E_T) \text{ kev} = \frac{\delta E}{\delta F} \Delta F$$

where $\delta E / \delta F = 0.3575 \text{ kev/kc.}$ for ΔF in kilocycles and a proton energy of 1982 kev. Target thickness is taken as the value of the proton energy difference between the peak and threshold ($E_p - E_T$) with the counter at 39.4 inches in accordance with equation (14), since the correction term (K) is negligible in this case. Entering Figure 1 with the value of target thickness ($\Delta E = 6.44 \text{ kev}$), one obtains a value of θ_{eff} for each value of $E_p - E_T$. The results follow:

Distance	Wavelength	Ratio	Ratio	Ratio	Ratio
35.1"	1	1.00	1.00	1.00	1.00
18.0	2.4	1.50	1.50	1.50	1.50
13.0	0.41	3.30	3.30	3.30	3.30
7.5	11.61	7.90	7.90	7.90	7.90

The above values are given in the form of the ratio of the distance of the wave from the surface of the crystal to the distance of the wave from the surface of the crystal.

The distance between the surface of the crystal and the surface of the crystal is given in the form of the ratio of the distance of the wave from the surface of the crystal to the distance of the wave from the surface of the crystal.

$$(31) \quad \frac{1}{2} \left(\frac{1}{2} + \frac{1}{2} \right) = \frac{1}{2}$$

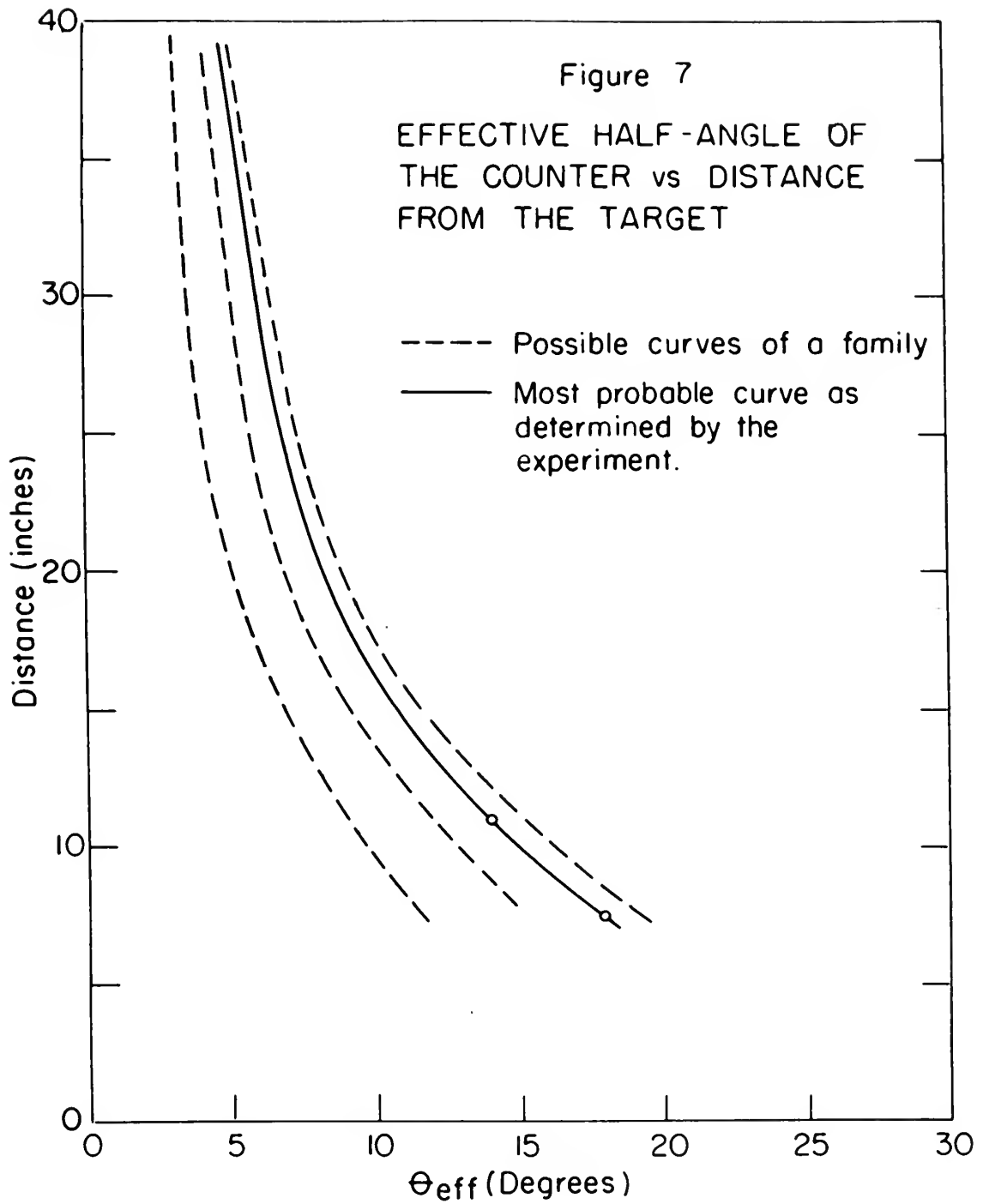
where $\frac{1}{2} \left(\frac{1}{2} + \frac{1}{2} \right) = \frac{1}{2}$ is the ratio of the distance of the wave from the surface of the crystal to the distance of the wave from the surface of the crystal. The above values are given in the form of the ratio of the distance of the wave from the surface of the crystal to the distance of the wave from the surface of the crystal.

<u>Distance from Target</u>	<u>$\Delta F(\text{ke})$</u>	<u>$E_p - E_T$</u>	<u>θ_{eff}</u>
39.4"	18	6.44 = ΔE	-
18.0	18	6.44	-
11.0	19	6.30	14°
7.5	20	7.15	18°

The values, $\theta_{\text{eff}} = 14^\circ, 18^\circ$ are plotted on the family of curves in Figure 7. The most probable curve is now drawn through these two points, giving the best value of the effective half-angle of this particular counter for any counter position between 7.5 and 39.4 inches. This curve may be extrapolated to some extent in either direction.

The following table may be of assistance in applying the above results to a long counter with paraffin diameter slightly different from the 7.5-inch diameter used in this laboratory.

<u>Distance (d)</u>	<u>$\tan \theta$ $= 3.75/d$</u>	<u>θ (Actual)</u>	<u>θ_{eff}</u>
39.4"	.09525	5.43°	4.6°
18.0	.20833	11.77°	9.0°
11.0	.34091	18.82°	14.0°
7.5	.50000	26.57°	18.0°



X. APPLICATION OF EQUATION (11)

The determination of target thickness, as given by equation (11), consists of four steps:

A. Determination of the effective half-angle of the counter. This value may be taken directly from Figure 7 when using the long counter employed in this laboratory.

B. Determination of the reaction threshold by either

1. Extrapolating the lower linear portion of the curve to the axis; or

2. A supplementary experiment using a target of about 20-kev thickness and an effective half-angle of about 4 degrees after the method described in Section VII. This method is useful only if the proton energy is very closely controlled and measured so that measurements of the threshold value may be duplicated within very narrow limits.

C. Measurement of the energy difference between peak and threshold ($E_p - E_t$) directly from the experimental yield curve.

D. Enter Figure 1 with the appropriate values of θ_{eff} and $E_p - E_t$ to get target thickness directly. For endoergic reactions other than the $Li(p,n)He^7$ reaction, target thickness must be calculated by equation (11).

One may also use Figures 1 and 7 for selecting a suitable counter position, assuming that a rough estimate of target thickness is available, either from past measurements of thickness or by visual

1. APPLICATION OF EQUATION (1)

The determination of target thickness, as given by equation

(1), consists of four steps:

A. Determination of the effective half-thickness of the counter. This value may be taken directly from Figure 1 when using the long counter referred to in the caption.

B. Determination of the reaction threshold by either

1. Approximation of the linear portion of the

curve to the origin.

2. A comparison of the count rate with a target of

about 30-mil thickness and an effective half-thickness of about 6 mils.

These values are then plotted on Figure 1. This method is

usually used when the reaction energy is near 0 and is complicated and

inconvenient when the reaction energy is of the order of 100 mils.

C. Determination of the reaction energy.

D. Determination of the energy of the reaction between

the target and the incident particle. This is done by using the

relationship between the reaction energy and the energy of the

incident particle. This is done by using the relationship between

the reaction energy and the energy of the incident particle.

The final result is the target thickness.

The final result is the target thickness.

The final result is the target thickness.

The final result is the target thickness.

inspection. In this connection, the lower dashed portions of the curves in Figure 1 should be avoided, as the peak position is not very sensitive to target thickness in these regions.

inspection. In this connection, the fact that the
evidence in this case is circumstantial, and the fact that it is
very sensitive to target disclosure in the past.

XI. CONCLUSIONS AND RECOMMENDATIONS

As a result of this study, it is concluded that the most accurate value of target thickness will be obtained by fitting a theoretical yield curve to the experimentally determined curve as described by Bonner and Butler³ and discussed herein. However, the method described in this thesis should give very good approximations of target thickness with but little effort from the very manner in which the effective half-angle of the counter has been assigned. The ability to measure the target thickness by the rise method for various counter positions is certainly an advantage as higher counting rates can be obtained with the use of a lower beam current; hence, the effects of ageing of the target while measuring the thickness will be decreased. This could be important when using thin targets, as they are subject to considerable thickening after only a few hours of use (Hinchey, Preston, and Stilson, unpublished data).

The information presented in Section VII dealing with proton energy resolution leads to the conclusion that this method cannot be expected to give nearly so good results for thicknesses of less than two kilovolts as would be expected for the range from two to twenty kilovolts.

To extend the results presented in this thesis, the following recommendations are made:

- A. Perform the experiment outlined in Section IX in order to:
1. Obtain a more accurate value of effective counter half-angle if possible.
 2. Determine whether the effective half-angle varies with target thickness. Any such dependence should be small and could be presented as a family of curves similar to the curve of effective half-angle versus distance of the counter from the target (Figure 1).
 3. Check the accuracy of the results presented herein for various counter positions and target thicknesses.
- B. Perform a series of experiments to determine the difference between the actual threshold and the apparent threshold as determined by a linear extrapolation to the axis. This information would be particularly helpful when using very thin targets (less than 2 kev).
- C. Determine the optimum dimensions and properties of a counter to be used specifically for measuring target thickness. In this connection, consider the fact that the counter will be used for endoergic reactions other than the $\text{Li}^7(p,n)\text{Be}^7$ reaction.
- D. Make calculations for the $\text{T}^3(p,n)\text{He}^3$ reaction similar to those presented here for the $\text{Li}^7(p,n)\text{Be}^7$ reaction, inasmuch as tritium targets are commonly used in this laboratory as a neutron source.

4. Perform the experiment outlined in Section IX

In order to:

1. Obtain a more accurate value of effective count

for half-angle if possible.

2. Determine whether the effective half-angle

varies with target thickness. Any such dependence should be small

and could be represented as a family of curves similar to the curve

of effective half-angle versus distance of the counter from the tar-

get (Figure 1).

3. Check the accuracy of the results presented

herein for various counter positions and target thicknesses.

5. Perform a series of experiments to determine the

difference between the actual thickness and the apparent thickness

as determined by a linear extrapolation to the axis. This differen-

tion would be important in the design of counter tubes.

(Less than 2 mm).

6. Determine the optimum distance and orientation of

a counter to be used specifically for measuring target thickness.

In this connection, determine the best the counter will be used

for end-on measurements of thin targets (0.1 mm thickness).

7. This experiment is the same as the one in Section 4 with

to those recorded here for the 117 source, the results should be

triple checked by the results used in the design of a counter

source.

APPENDIX A

DERIVATION OF G

In deriving the expressions for the fraction G of neutrons from an elemental thickness of target which enters the counter, Figure 8 applies throughout. The basic assumption is that the neutrons are emitted isotropically in the center-of-mass system. Sphere A represents the locus of neutron velocity vectors (V_n) in the center-of-mass system. The velocity of the center of mass (V_{cm}) is then added to each point of sphere A to obtain sphere B which is then the locus of neutron velocity vectors in the laboratory system. The counter subtends a solid angle at the target in the form of a cone, the half-angle (half of the apex angle) of which is designated by θ . If sphere B lies entirely within this cone, then all of the neutrons will strike the counter. If the velocity of the center of mass is less than the neutron velocity in the center-of-mass system, then neutrons from an area A_H on the sphere B will strike the counter. For the situation intermediate between these two, there will be a high-energy group of neutrons from area A_H and a lower-energy group from area A_L intercepted by the counter. The number of neutrons in the groups has the same ratio to the total number of neutrons as the area in question bears to the total area of the sphere.

APPENDIX A

EXPLANATION OF

In deriving the explanation for the reaction of the system

from an element, a balance of forces will be shown and used as

Figure 8 explains the point. The same conclusion is the the

members are subjected to the same force and the same

Figure 8 represents the force of reaction velocity vector V_r

in the center-of-mass system. The velocity of the center of mass

(V_{cm}) is then added to each point to obtain the velocity of

which is then the force of reaction velocity vector in the center-

of-mass system. The center of mass is at the center of

the form of a ring, the velocity vector of the center of mass

which is indicated by V_{cm} in Figure 8. The velocity of the

center, the velocity of the reaction will be the same as the

velocity of the center of mass. The velocity of the center of mass

in the center-of-mass system. The velocity of the center of mass

which is indicated by V_{cm} in Figure 8. The velocity of the

center, the velocity of the reaction will be the same as the

velocity of the center of mass. The velocity of the center of mass

which is indicated by V_{cm} in Figure 8. The velocity of the

center, the velocity of the reaction will be the same as the

velocity of the center of mass. The velocity of the center of mass

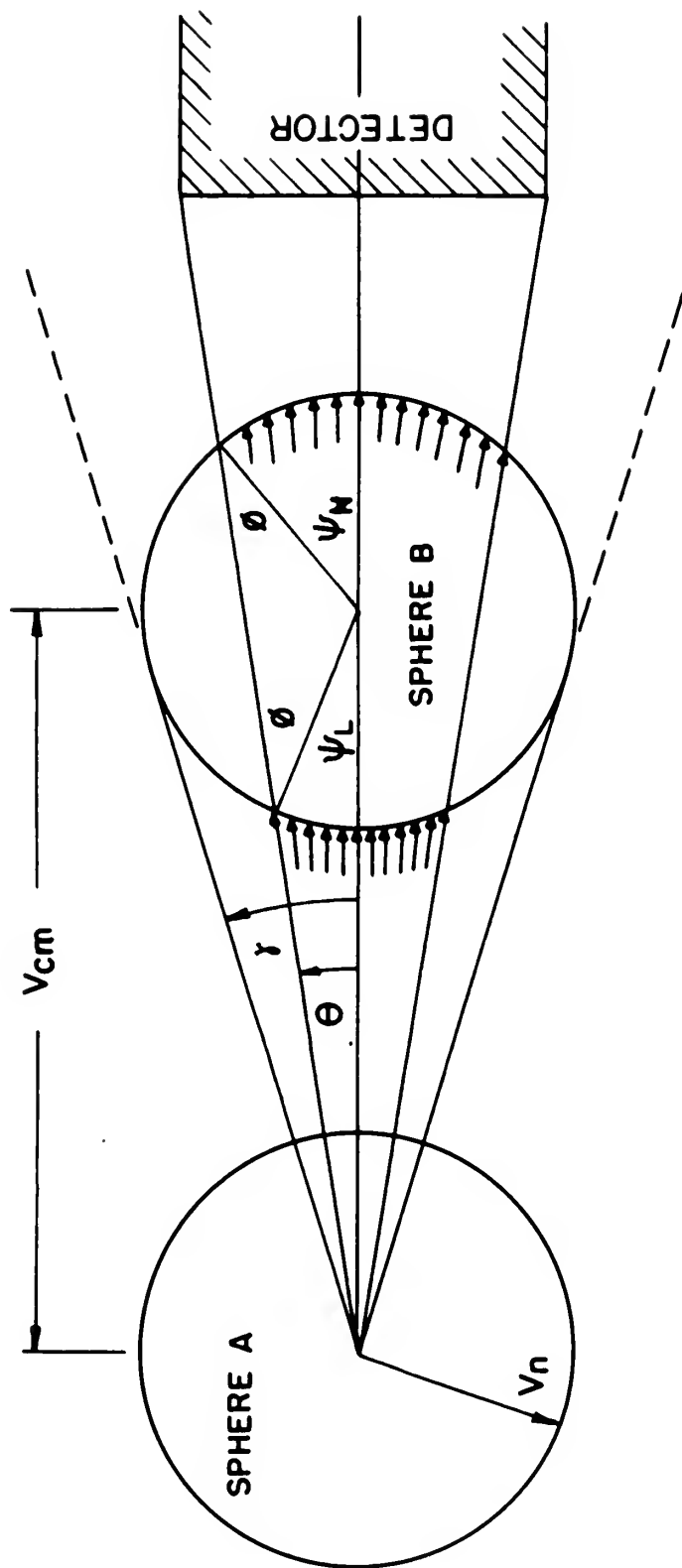


Figure 8
SCHEMATIC REPRESENTATION OF THE MECHANICS
OF AN ENDOERGIC REACTION JUST ABOVE THRESHOLD

The following summary is given to clarify the three situations:

- E = proton energy (instantaneous)
- E_0 = proton energy (incident upon the target)
- E_T = proton energy (threshold)
- E_c = proton energy ($\gamma = 0$)
- E_L = proton energy ($\gamma = \pi/2$; $V_n = V_{cm}$)
- G = fraction of neutrons entering the counter.

<u>G</u>	<u>E</u>	<u>γ</u>	<u>Remarks</u>
G_1	$E_T < E < E_c$	$\gamma < 0$	All neutrons strike counter
G_2	$E_c < E < E_L$	$0 < \gamma < \pi/2$	Two groups
G_3	$E_L < E$	$\gamma = \pi$	One group

- V_n = neutron velocity in center-of-mass system
- V_{cm} = velocity of center of mass
- $A_{H,L}$ = area on surface of sphere B through which pass the high- and low-energy groups of neutrons respectively, which are intercepted by the counter.

The following energy levels are given in eV:

Approximate

- 1. 2. 3. 4. 5. 6. 7. 8. 9. 10. 11. 12. 13. 14. 15. 16. 17. 18. 19. 20. 21. 22. 23. 24. 25. 26. 27. 28. 29. 30. 31. 32. 33. 34. 35. 36. 37. 38. 39. 40. 41. 42. 43. 44. 45. 46. 47. 48. 49. 50. 51. 52. 53. 54. 55. 56. 57. 58. 59. 60. 61. 62. 63. 64. 65. 66. 67. 68. 69. 70. 71. 72. 73. 74. 75. 76. 77. 78. 79. 80. 81. 82. 83. 84. 85. 86. 87. 88. 89. 90. 91. 92. 93. 94. 95. 96. 97. 98. 99. 100.

| 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 | 21 | 22 | 23 | 24 | 25 | 26 | 27 | 28 | 29 | 30 | 31 | 32 | 33 | 34 | 35 | 36 | 37 | 38 | 39 | 40 | 41 | 42 | 43 | 44 | 45 | 46 | 47 | 48 | 49 | 50 | 51 | 52 | 53 | 54 | 55 | 56 | 57 | 58 | 59 | 60 | 61 | 62 | 63 | 64 | 65 | 66 | 67 | 68 | 69 | 70 | 71 | 72 | 73 | 74 | 75 | 76 | 77 | 78 | 79 | 80 | 81 | 82 | 83 | 84 | 85 | 86 | 87 | 88 | 89 | 90 | 91 | 92 | 93 | 94 | 95 | 96 | 97 | 98 | 99 | 100 |
|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|
| 0.00 | 0.01 | 0.02 | 0.03 | 0.04 | 0.05 | 0.06 | 0.07 | 0.08 | 0.09 | 0.10 | 0.11 | 0.12 | 0.13 | 0.14 | 0.15 | 0.16 | 0.17 | 0.18 | 0.19 | 0.20 | 0.21 | 0.22 | 0.23 | 0.24 | 0.25 | 0.26 | 0.27 | 0.28 | 0.29 | 0.30 | 0.31 | 0.32 | 0.33 | 0.34 | 0.35 | 0.36 | 0.37 | 0.38 | 0.39 | 0.40 | 0.41 | 0.42 | 0.43 | 0.44 | 0.45 | 0.46 | 0.47 | 0.48 | 0.49 | 0.50 | 0.51 | 0.52 | 0.53 | 0.54 | 0.55 | 0.56 | 0.57 | 0.58 | 0.59 | 0.60 | 0.61 | 0.62 | 0.63 | 0.64 | 0.65 | 0.66 | 0.67 | 0.68 | 0.69 | 0.70 | 0.71 | 0.72 | 0.73 | 0.74 | 0.75 | 0.76 | 0.77 | 0.78 | 0.79 | 0.80 | 0.81 | 0.82 | 0.83 | 0.84 | 0.85 | 0.86 | 0.87 | 0.88 | 0.89 | 0.90 | 0.91 | 0.92 | 0.93 | 0.94 | 0.95 | 0.96 | 0.97 | 0.98 | 0.99 | 1.00 |

The following energy levels are given in eV:

Approximate

1. 2. 3. 4. 5. 6. 7. 8. 9. 10. 11. 12. 13. 14. 15. 16. 17. 18. 19. 20. 21. 22. 23. 24. 25. 26. 27. 28. 29. 30. 31. 32. 33. 34. 35. 36. 37. 38. 39. 40. 41. 42. 43. 44. 45. 46. 47. 48. 49. 50. 51. 52. 53. 54. 55. 56. 57. 58. 59. 60. 61. 62. 63. 64. 65. 66. 67. 68. 69. 70. 71. 72. 73. 74. 75. 76. 77. 78. 79. 80. 81. 82. 83. 84. 85. 86. 87. 88. 89. 90. 91. 92. 93. 94. 95. 96. 97. 98. 99. 100.

$$\frac{V_{cm}}{\sin(\pi - \phi)} = \frac{V_n}{\sin \theta} \quad \text{or} \quad \frac{V_{cm}}{\sin \phi} = \frac{V_n}{\sin \theta}$$

Therefore $\sin \phi = \frac{V_{cm}}{V_n} \sin \theta$

$$\cos \phi = \left[1 - \left(\frac{V_{cm}}{V_n} \right)^2 \sin^2 \theta \right]^{1/2}$$

Area: $A_{H,L} = 2\pi V_n^2 (1 - \cos \psi_{H,L})$

$$\cos \psi_H = \cos(\phi + \theta) = \cos \phi \cos \theta - \sin \phi \sin \theta$$

$$= \cos \theta \left[1 - \left(\frac{V_{cm}}{V_n} \right)^2 \sin^2 \theta \right]^{1/2} - \frac{V_{cm}}{V_n} \sin^2 \theta$$

$$\cos \psi_L = \cos(\phi - \theta) = \cos \phi \cos \theta + \sin \phi \sin \theta$$

$$= \cos \theta \left[1 - \left(\frac{V_{cm}}{V_n} \right)^2 \sin^2 \theta \right]^{1/2} + \frac{V_{cm}}{V_n} \sin^2 \theta$$

$$G = \frac{A_H + \delta A_L}{A_{\text{sphere}}} \quad \text{where} \quad \begin{cases} \delta = 1 & \text{for } V_{cm} > V_n \\ \delta = 0 & \text{for } V_{cm} < V_n \end{cases}$$

Therefore

$$\frac{V_1}{V_2} = \frac{V_3}{V_4} \quad \text{or} \quad \frac{V_1}{V_2} = \frac{V_3}{V_4}$$

$$\text{Therefore } \frac{V_1}{V_2} = \frac{V_3}{V_4}$$

$$\cos \theta = \left[1 - \left(\frac{V_1}{V_2} \right)^2 \right]^{1/2}$$

$$\text{Area} = \frac{1}{2} V_1 V_2 \sin \theta$$

$$\text{cos } \theta = \left[1 - \left(\frac{V_1}{V_2} \right)^2 \right]^{1/2}$$

$$\text{Area} = \frac{1}{2} V_1 V_2 \left[1 - \left(\frac{V_1}{V_2} \right)^2 \right]^{1/2}$$

$$\text{cos } \theta = \left[1 - \left(\frac{V_1}{V_2} \right)^2 \right]^{1/2}$$

$$\text{Area} = \frac{1}{2} V_1 V_2 \left[1 - \left(\frac{V_1}{V_2} \right)^2 \right]^{1/2}$$

$$\frac{V_1}{V_2} = \frac{V_3}{V_4}$$

Therefore

$$\left\{ a \sin \frac{2\pi y}{\lambda} + \left[a \sin \left(\frac{2\pi y}{\lambda} - t \right) \cos \theta - t \right] \right\} \sin t = 0$$

$$\left\{ a \sin \frac{2\pi y}{\lambda} + \left[a \sin \left(\frac{2\pi y}{\lambda} - t \right) \cos \theta - t \right] \right\} \cos t = 0$$

$$\sin t = 0 \quad (251)$$

$$\cos t = 0 \quad (252)$$

$$\left\{ a \sin \frac{2\pi y}{\lambda} + \left[a \sin \left(\frac{2\pi y}{\lambda} - t \right) \cos \theta - t \right] \right\} \sin t = 0 \quad (253)$$

$$\left\{ a \sin \frac{2\pi y}{\lambda} + \left[a \sin \left(\frac{2\pi y}{\lambda} - t \right) \cos \theta - t \right] \right\} \cos t = 0$$

$$\sin t = 0 \quad (254)$$

$$\cos t = 0 \quad (255)$$

$$\sin t = 0 \quad (256)$$

$$\cos t = 0 \quad (257)$$

$$\sin t = 0 \quad (258)$$

$$\cos t = 0 \quad (259)$$

$$\sin t = 0 \quad (260)$$

$$\cos t = 0 \quad (261)$$

$$(33) \quad v_{cm} = \frac{m_1}{m_1 + m_2} v_1 = \frac{(2m_1 E)^{1/2}}{m_1 + m_2}$$

$$(34) \quad v_n = \left\{ \frac{2m_2 m_1}{m_3 (m_1 + m_2)^2} \left[E + \frac{(m_1 + m_2)}{m_2} Q \right] \right\}^{1/2} \quad (\text{reference 1})$$

$$\text{But } \frac{m_1 + m_2}{m_2} Q = -E_T$$

$$(35) \quad v_n = \left\{ \frac{2m_2 m_1}{m_3 (m_1 + m_2)^2} (E - E_T) \right\}^{1/2} = \text{const } (E - E_T)^{1/2}$$

$$\left(\frac{v_{cm}}{v_n} \right)^2 = \frac{m_1 m_3}{m_2 m_1} \left(\frac{E}{E - E_T} \right)$$

Hence, the expressions for $G_{1,2,3}$ may be written:

$$(36a) \quad G_1 = 1 \quad E < E_c$$

$$(36b) \quad G_2 = 1 - \cos \theta \left[1 - \frac{m_1 m_3}{m_2 m_1} \frac{E \sin^2 \theta}{(E - E_T)} \right]^{1/2} \quad . E_c < E < E_L$$

$$(36c) \quad G_3 = 1/2 \left\{ 1 - \cos \theta \left[1 - \frac{m_1 m_3}{m_2 m_1} \frac{E \sin^2 \theta}{(E - E_T)} \right]^{1/2} + \sin^2 \theta \left[\frac{m_1 m_3}{m_2 m_1} \frac{E}{(E - E_T)} \right]^{1/2} \right\} \quad E > E_L$$

$$\frac{1}{\sqrt{1-\beta^2}} \left(\frac{1}{\sqrt{1-\beta^2}} \right) = \frac{1}{\sqrt{1-\beta^2}} \quad (33)$$

$$\frac{1}{\sqrt{1-\beta^2}} \left\{ \left[\frac{1}{\sqrt{1-\beta^2}} \right] \right\} = \quad (34)$$

$$\frac{1}{\sqrt{1-\beta^2}} = \frac{1}{\sqrt{1-\beta^2}}$$

$$\frac{1}{\sqrt{1-\beta^2}} = \frac{1}{\sqrt{1-\beta^2}} \quad (35)$$

$$\frac{1}{\sqrt{1-\beta^2}} = \frac{1}{\sqrt{1-\beta^2}}$$

$$\frac{1}{\sqrt{1-\beta^2}} = \frac{1}{\sqrt{1-\beta^2}} \quad (36)$$

$$\frac{1}{\sqrt{1-\beta^2}} = \frac{1}{\sqrt{1-\beta^2}} \quad (37)$$

$$\frac{1}{\sqrt{1-\beta^2}} = \frac{1}{\sqrt{1-\beta^2}} \quad (38)$$

$$\frac{1}{\sqrt{1-\beta^2}} = \frac{1}{\sqrt{1-\beta^2}} \quad (39)$$

$$\frac{1}{\sqrt{1-\beta^2}} = \frac{1}{\sqrt{1-\beta^2}}$$

If E_0 is defined as the proton energy at which the solid angle of the neutron cone is equal to the solid angle subtended at the target by the counter (i.e., $\gamma = \theta$ at $E = E_0$), then:

$$\sin \theta = \frac{V_n}{V_{cm}}$$

$$\sin^2 \theta = \left(\frac{V_n}{V_{cm}} \right)^2 = \frac{m_2 m_4}{m_1 m_3} \left(\frac{E_c - E_T}{E_c} \right)$$

$$(37) \quad E_c = \frac{E_T}{1 - \frac{m_1 m_3}{m_2 m_4} \sin^2 \theta}$$

E_c values are calculated for various θ (for the $Li(p,n)$ reaction only) in Table II, Appendix B, and presented as a curve in Figure 9.

$$(38) \quad \text{Let } k = \left(1 - \frac{m_1 m_3}{m_2 m_4} \sin^2 \theta \right)^{1/2} \cos \theta = \left(\frac{E_T}{E_c} \right)^{1/2} \cos \theta$$

$$(39) \quad \text{and } b = \left(\frac{m_1 m_3}{m_2 m_4} \right)^{1/2} \sin^2 \theta$$

The equations for $G_{1,2,3}$ become:

11/10/1918. In the morning, the sun was shining and the
 temperature was 60°. The wind was from the north and the
 clouds were light. The water was calm and the sky was blue.

— 2 —

5

(7E)

$$(40a) \quad G_1 = 1 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad E < E_c$$

$$(40b) \quad G_2 = 1 - k \left(\frac{E - E_c}{E - E_T} \right)^{1/2} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad E_c < E < E_L$$

$$(40c) \quad G_3 = 1/2 \left[1 - k \left(\frac{E - E_c}{E - E_T} \right)^{1/2} + b \left(\frac{E}{E - E_T} \right)^{1/2} \right] \quad . \quad E > E_L$$

where E_L is given by evaluating E_c at $\theta = \pi/2$ in equation (37)

above:

$$(41) \quad E_L = \frac{E_T}{1 - \frac{m_1 m_3}{m_2 m_4}}$$

APPENDIX B

COMPUTATIONS RELATIVE TO THE $\text{Li}^7(\text{p},\text{n})\text{Be}^7$ REACTION

The following values will be used in connection with the $\text{Li}^7(\text{p},\text{n})\text{Be}^7$ reaction:

$$E_T = 1.882 \text{ Mev}$$

$$Q = -1.6456 \text{ Mev}$$

$$m_1 = m(\text{p}) = 1.007593$$

$$m_2 = m(\text{Li}^7) = 7.01659$$

$$m_3 = m(\text{n}) = 1.008982$$

$$m_4 = m(\text{Be}^7) = 7.01697$$

The threshold and Q-values are those determined by Herb, Snowden, and Sala¹², while the nuclear mass values are from a compilation used in this laboratory (from numerous sources).

$$m_1 + m_2 = 8.024183$$

$$m_1 + m_3 = 1.0166432$$

$$m_2 + m_4 = 14.033562$$

$$\frac{m_1 + m_3}{m_2 + m_4} = 0.0206487$$

$$1 - \frac{m_1 + m_3}{m_2 + m_4} = 0.9793513$$

APPENDIX B

COMPUTATIONS RELATIVE TO THE $13(p, n)12n$ REACTION

The following values will be used in connection with the

$13(p, n)12n$ reaction:

$$\begin{aligned} \bar{v}_1 &= 1.888 \text{ ev} \\ \bar{v}_2 &= -1.1156 \text{ ev} \\ \bar{v}_3 &= 1.0076 \\ \bar{v}_4 &= 0.0157 \\ \bar{v}_5 &= 1.0090 \\ \bar{v}_6 &= 0.0157 \end{aligned}$$

the above values are used in the calculation of the
 $13(p, n)12n$ reaction, which is the most important one from a com-
 putational point of view.

$$\begin{aligned} \bar{v}_1 &= 1.888 \\ \bar{v}_2 &= -1.1156 \\ \bar{v}_3 &= 1.0076 \\ \bar{v}_4 &= 0.0157 \\ \bar{v}_5 &= 1.0090 \\ \bar{v}_6 &= 0.0157 \end{aligned}$$

$$\bar{v}_1 = 1.888$$

$$(42) \quad E_L = \frac{E_T}{1 - \frac{m_1 m_3}{m_2 m_4}} = 1921.68 \text{ kev} = E_T + 39.68 \text{ kev.}$$

$$(43) \quad E_c = \frac{E_T}{1 - \frac{m_1 m_3}{m_2 m_4} \sin^2 \theta} = \frac{1882}{1 - 0.0206487 \sin^2 \theta} \text{ kev.}$$

Equation (43) is used for computing E_c (Table II) for effective half-angles of the counter up to 30 degrees, and the results are given as the curve of Figure 9.

Values of k for various half-angles of the counter are calculated from the relation (equation 38):

$$k = (1 - \frac{m_1 m_3}{m_2 m_4} \sin^2 \theta)^{1/2} \cos \theta = (\frac{E_T}{E_c})^{1/2} \cos \theta.$$

These k values are given in Table III for later use in other computations.

In Table IV is the computed value of

$$\frac{4(E_c - E_T)}{k^2 + 1/k^2 - 2}$$

used in Section VI for determining the region of the geometric peak.

(12)
$$S^2 = \frac{1}{n} \sum_{i=1}^n (x_i - \bar{x})^2$$

(13)
$$S^2 = \frac{1}{n} \sum_{i=1}^n x_i^2 - \bar{x}^2$$

Equation (12) is used for computing the sample variance. The sample variance is the average of the squared deviations from the mean. Equation (13) is used for computing the sample variance when the mean is known. The sample variance is the average of the squared deviations from the mean.

$$S^2 = \frac{1}{n} \sum_{i=1}^n x_i^2 - \bar{x}^2$$

These two equations are used for computing the sample variance. Equation (12) is used for computing the sample variance when the mean is unknown. Equation (13) is used for computing the sample variance when the mean is known.

$$S^2 = \frac{1}{n} \sum_{i=1}^n x_i^2 - \bar{x}^2$$

These two equations are used for computing the sample variance. Equation (12) is used for computing the sample variance when the mean is unknown. Equation (13) is used for computing the sample variance when the mean is known.

Tables V (A-G) includes the calculations for the correction term K and target thickness ΔE for various values of the proton energy difference between the geometric peak and threshold ($E_p - E_T$) according to equation (11):

$$(11) \quad \Delta E = E_p - E_T - K$$

$$\text{where } K = \left[(E_p - E_T)^{1/2} - k(E_p - E_c)^{1/2} \right]^2$$

Curves of ΔE versus ($E_p - E_T$) for values of θ_{eff} from 4 to 28 degrees are constructed from the results of Table V and presented as Figure 1.

Table 1 (A-B) indicates the calculated values of the constant

term k and target difference Δ for various values of α and β .

Figure 2 shows the variation of the constant term k with α and β .

According to equation (11):

$$k = \frac{1}{\alpha} \left(\frac{1}{\beta} - 1 \right) \quad (11)$$

$$\text{where } k = \frac{1}{\alpha} \left(\frac{1}{\beta} - 1 \right) \quad \text{and } \Delta = \frac{1}{\alpha} \left(\frac{1}{\beta} - 1 \right)$$

Table 1 (A-B) indicates the calculated values of the constant

term k and target difference Δ for various values of α and β .

Figure 2 shows the variation of the constant term k with α and β .

TABLE II

Evaluation of E_c for various effective half-angles of the counter
(Equations 37 and 43)

| θ | $\sin^2 \theta$ | $\frac{.0206487}{\sin^2 \theta}$ | E_T/E_c | E_c (kev) |
|----------|-----------------|----------------------------------|-----------|-------------|
| 2 | .001218 | .000025 | .999975 | 1882.047 |
| 3 | .002739 | .000057 | .999943 | 1882.107 |
| 4 | .004866 | .000100 | .999900 | 1882.188 |
| 5 | .007597 | .000157 | .999843 | 1882.295 |
| 6 | .010927 | .000226 | .999774 | 1882.425 |
| 7 | .014852 | .000307 | .999693 | 1882.577 |
| 8 | .019368 | .000400 | .999600 | 1882.753 |
| 9 | .024470 | .000505 | .999495 | 1882.950 |
| 10 | .030154 | .000623 | .999377 | 1883.173 |
| 11 | .036408 | .000752 | .999248 | 1883.416 |
| 12 | .043227 | .000893 | .999107 | 1883.682 |
| 13 | .050603 | .001045 | .998955 | 1883.968 |
| 14 | .05853 | .001209 | .998791 | 1884.278 |
| 16 | .07598 | .001569 | .998431 | 1884.957 |
| 18 | .09549 | .001972 | .998028 | 1885.718 |
| 20 | .11698 | .002415 | .997585 | 1886.556 |
| 22 | .14033 | .002898 | .997102 | 1887.469 |
| 24 | .16544 | .003416 | .996584 | 1888.450 |
| 26 | .19217 | .003968 | .996032 | 1889.497 |
| 28 | .22040 | .004551 | .995449 | 1890.604 |
| 30 | .25000 | .005162 | .994838 | 1891.765 |

二、三、四

Investigation of the various types of vegetation in the various parts of the island.

(C. 128 TE univ. de p'd)

| Year | 1900 | 1901 | 1902 | 1903 | 1904 |
|------|---------|---------|---------|---------|---------|
| 1900 | 1000000 | 1000000 | 1000000 | 1000000 | 1000000 |
| 1901 | 1000000 | 1000000 | 1000000 | 1000000 | 1000000 |
| 1902 | 1000000 | 1000000 | 1000000 | 1000000 | 1000000 |
| 1903 | 1000000 | 1000000 | 1000000 | 1000000 | 1000000 |
| 1904 | 1000000 | 1000000 | 1000000 | 1000000 | 1000000 |

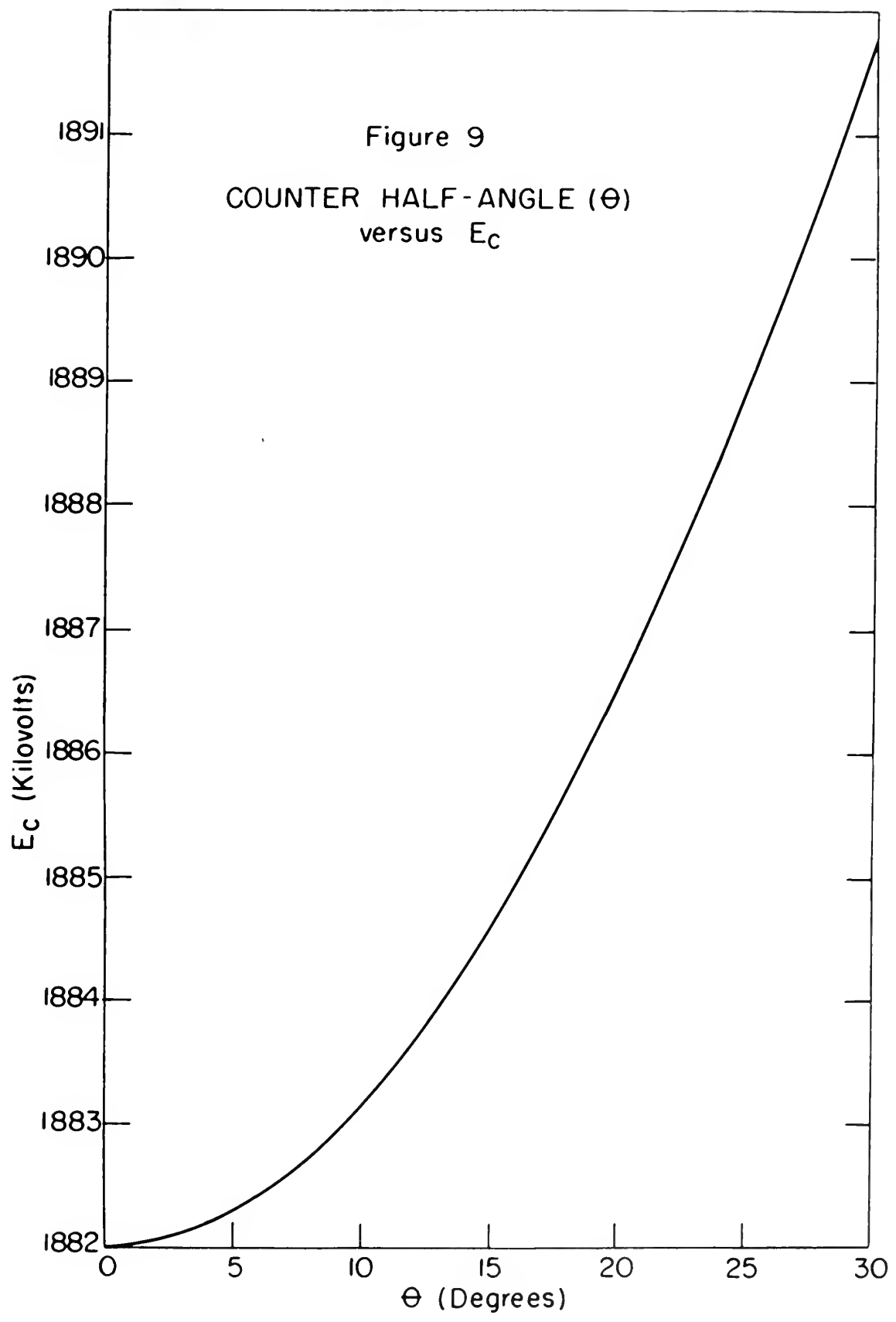


TABLE III

Evaluation of k as given by Equation (38)

| θ° | $E_c - E_T$ | (E_T/E_c) | $(E_T/E_c)^{1/2}$ | $\cos \theta$ | k |
|----------------|-------------|-------------|-------------------|---------------|--------|
| 2 | .047 | .99998 | .99999 | .99939 | .99938 |
| 3 | .107 | .99994 | .99997 | .99863 | .99860 |
| 4 | .188 | .99990 | .99995 | .99756 | .99751 |
| 5 | .295 | .99984 | .99992 | .99619 | .99611 |
| 6 | .425 | .99977 | .99988 | .99452 | .99441 |
| 7 | .577 | .99969 | .99984 | .99255 | .99239 |
| 8 | .753 | .99960 | .99980 | .99027 | .99007 |
| 9 | .950 | .99950 | .99975 | .98769 | .98744 |
| 10 | 1.173 | .99938 | .99969 | .98481 | .98450 |
| 11 | 1.416 | .99925 | .99962 | .98163 | .98126 |
| 12 | 1.682 | .99911 | .99955 | .97815 | .97771 |
| 13 | 1.968 | .99896 | .99948 | .97437 | .97386 |
| 14 | 2.278 | .99879 | .99940 | .97030 | .96972 |
| 16 | 2.957 | .99843 | .99921 | .96126 | .96050 |
| 18 | 3.718 | .99803 | .99901 | .95106 | .95012 |
| 20 | 4.556 | .99759 | .99879 | .93969 | .93855 |
| 22 | 5.469 | .99710 | .99855 | .92713 | .92584 |
| 24 | 6.450 | .99658 | .99829 | .91355 | .91199 |
| 26 | 7.497 | .99603 | .99801 | .89879 | .89700 |
| 28 | 8.604 | .99545 | .99772 | .88295 | .88094 |
| 30 | 9.765 | .99484 | .99742 | .86603 | .86380 |

TABLE III

(The number of cases in each group)

| <u>Age</u> | <u>Sex</u> | <u>Occupation</u> | <u>Education</u> | <u>Income</u> | <u>No.</u> |
|------------|------------|-------------------|------------------|---------------|------------|
| 15-20 | Male | Student | High School | \$10. | 5 |
| 21-25 | Female | Homemaker | High School | \$10. | 5 |
| 26-30 | Male | Student | College | \$15. | 5 |
| 31-35 | Female | Homemaker | College | \$20. | 5 |
| 36-40 | Male | Student | College | \$25. | 5 |
| 41-45 | Female | Homemaker | College | \$30. | 5 |
| 46-50 | Male | Student | College | \$35. | 5 |
| 51-55 | Female | Homemaker | College | \$40. | 5 |
| 56-60 | Male | Student | College | \$45. | 5 |
| 61-65 | Female | Homemaker | College | \$50. | 5 |
| 66-70 | Male | Student | College | \$55. | 5 |
| 71-75 | Female | Homemaker | College | \$60. | 5 |
| 76-80 | Male | Student | College | \$65. | 5 |
| 81-85 | Female | Homemaker | College | \$70. | 5 |
| 86-90 | Male | Student | College | \$75. | 5 |
| 91-95 | Female | Homemaker | College | \$80. | 5 |
| 96-100 | Male | Student | College | \$85. | 5 |
| 101-105 | Female | Homemaker | College | \$90. | 5 |
| 106-110 | Male | Student | College | \$95. | 5 |
| 111-115 | Female | Homemaker | College | \$100. | 5 |
| 116-120 | Male | Student | College | \$105. | 5 |
| 121-125 | Female | Homemaker | College | \$110. | 5 |
| 126-130 | Male | Student | College | \$115. | 5 |
| 131-135 | Female | Homemaker | College | \$120. | 5 |
| 136-140 | Male | Student | College | \$125. | 5 |
| 141-145 | Female | Homemaker | College | \$130. | 5 |
| 146-150 | Male | Student | College | \$135. | 5 |
| 151-155 | Female | Homemaker | College | \$140. | 5 |
| 156-160 | Male | Student | College | \$145. | 5 |
| 161-165 | Female | Homemaker | College | \$150. | 5 |
| 166-170 | Male | Student | College | \$155. | 5 |
| 171-175 | Female | Homemaker | College | \$160. | 5 |
| 176-180 | Male | Student | College | \$165. | 5 |
| 181-185 | Female | Homemaker | College | \$170. | 5 |
| 186-190 | Male | Student | College | \$175. | 5 |
| 191-195 | Female | Homemaker | College | \$180. | 5 |
| 196-200 | Male | Student | College | \$185. | 5 |
| 201-205 | Female | Homemaker | College | \$190. | 5 |
| 206-210 | Male | Student | College | \$195. | 5 |
| 211-215 | Female | Homemaker | College | \$200. | 5 |
| 216-220 | Male | Student | College | \$205. | 5 |
| 221-225 | Female | Homemaker | College | \$210. | 5 |
| 226-230 | Male | Student | College | \$215. | 5 |
| 231-235 | Female | Homemaker | College | \$220. | 5 |
| 236-240 | Male | Student | College | \$225. | 5 |
| 241-245 | Female | Homemaker | College | \$230. | 5 |
| 246-250 | Male | Student | College | \$235. | 5 |
| 251-255 | Female | Homemaker | College | \$240. | 5 |
| 256-260 | Male | Student | College | \$245. | 5 |
| 261-265 | Female | Homemaker | College | \$250. | 5 |
| 266-270 | Male | Student | College | \$255. | 5 |
| 271-275 | Female | Homemaker | College | \$260. | 5 |
| 276-280 | Male | Student | College | \$265. | 5 |
| 281-285 | Female | Homemaker | College | \$270. | 5 |
| 286-290 | Male | Student | College | \$275. | 5 |
| 291-295 | Female | Homemaker | College | \$280. | 5 |
| 296-300 | Male | Student | College | \$285. | 5 |
| 301-305 | Female | Homemaker | College | \$290. | 5 |
| 306-310 | Male | Student | College | \$295. | 5 |
| 311-315 | Female | Homemaker | College | \$300. | 5 |
| 316-320 | Male | Student | College | \$305. | 5 |
| 321-325 | Female | Homemaker | College | \$310. | 5 |
| 326-330 | Male | Student | College | \$315. | 5 |
| 331-335 | Female | Homemaker | College | \$320. | 5 |
| 336-340 | Male | Student | College | \$325. | 5 |
| 341-345 | Female | Homemaker | College | \$330. | 5 |
| 346-350 | Male | Student | College | \$335. | 5 |
| 351-355 | Female | Homemaker | College | \$340. | 5 |
| 356-360 | Male | Student | College | \$345. | 5 |
| 361-365 | Female | Homemaker | College | \$350. | 5 |
| 366-370 | Male | Student | College | \$355. | 5 |
| 371-375 | Female | Homemaker | College | \$360. | 5 |
| 376-380 | Male | Student | College | \$365. | 5 |
| 381-385 | Female | Homemaker | College | \$370. | 5 |
| 386-390 | Male | Student | College | \$375. | 5 |
| 391-395 | Female | Homemaker | College | \$380. | 5 |
| 396-400 | Male | Student | College | \$385. | 5 |
| 401-405 | Female | Homemaker | College | \$390. | 5 |
| 406-410 | Male | Student | College | \$395. | 5 |
| 411-415 | Female | Homemaker | College | \$400. | 5 |
| 416-420 | Male | Student | College | \$405. | 5 |
| 421-425 | Female | Homemaker | College | \$410. | 5 |
| 426-430 | Male | Student | College | \$415. | 5 |
| 431-435 | Female | Homemaker | College | \$420. | 5 |
| 436-440 | Male | Student | College | \$425. | 5 |
| 441-445 | Female | Homemaker | College | \$430. | 5 |
| 446-450 | Male | Student | College | \$435. | 5 |
| 451-455 | Female | Homemaker | College | \$440. | 5 |
| 456-460 | Male | Student | College | \$445. | 5 |
| 461-465 | Female | Homemaker | College | \$450. | 5 |
| 466-470 | Male | Student | College | \$455. | 5 |
| 471-475 | Female | Homemaker | College | \$460. | 5 |
| 476-480 | Male | Student | College | \$465. | 5 |
| 481-485 | Female | Homemaker | College | \$470. | 5 |
| 486-490 | Male | Student | College | \$475. | 5 |
| 491-495 | Female | Homemaker | College | \$480. | 5 |
| 496-500 | Male | Student | College | \$485. | 5 |
| 501-505 | Female | Homemaker | College | \$490. | 5 |
| 506-510 | Male | Student | College | \$495. | 5 |
| 511-515 | Female | Homemaker | College | \$500. | 5 |
| 516-520 | Male | Student | College | \$505. | 5 |
| 521-525 | Female | Homemaker | College | \$510. | 5 |
| 526-530 | Male | Student | College | \$515. | 5 |
| 531-535 | Female | Homemaker | College | \$520. | 5 |
| 536-540 | Male | Student | College | \$525. | 5 |
| 541-545 | Female | Homemaker | College | \$530. | 5 |
| 546-550 | Male | Student | College | \$535. | 5 |
| 551-555 | Female | Homemaker | College | \$540. | 5 |
| 556-560 | Male | Student | College | \$545. | 5 |
| 561-565 | Female | Homemaker | College | \$550. | 5 |
| 566-570 | Male | Student | College | \$555. | 5 |
| 571-575 | Female | Homemaker | College | \$560. | 5 |
| 576-580 | Male | Student | College | \$565. | 5 |
| 581-585 | Female | Homemaker | College | \$570. | 5 |
| 586-590 | Male | Student | College | \$575. | 5 |
| 591-595 | Female | Homemaker | College | \$580. | 5 |
| 596-600 | Male | Student | College | \$585. | 5 |
| 601-605 | Female | Homemaker | College | \$590. | 5 |
| 606-610 | Male | Student | College | \$595. | 5 |
| 611-615 | Female | Homemaker | College | \$600. | 5 |
| 616-620 | Male | Student | College | \$605. | 5 |
| 621-625 | Female | Homemaker | College | \$610. | 5 |
| 626-630 | Male | Student | College | \$615. | 5 |
| 631-635 | Female | Homemaker | College | \$620. | 5 |
| 636-640 | Male | Student | College | \$625. | 5 |
| 641-645 | Female | Homemaker | College | \$630. | 5 |
| 646-650 | Male | Student | College | \$635. | 5 |
| 651-655 | Female | Homemaker | College | \$640. | 5 |
| 656-660 | Male | Student | College | \$645. | 5 |
| 661-665 | Female | Homemaker | College | \$650. | 5 |
| 666-670 | Male | Student | College | \$655. | 5 |
| 671-675 | Female | Homemaker | College | \$660. | 5 |
| 676-680 | Male | Student | College | \$665. | 5 |
| 681-685 | Female | Homemaker | College | \$670. | 5 |
| 686-690 | Male | Student | College | \$675. | 5 |
| 691-695 | Female | Homemaker | College | \$680. | 5 |
| 696-700 | Male | Student | College | \$685. | 5 |
| 701-705 | Female | Homemaker | College | \$690. | 5 |
| 706-710 | Male | Student | College | \$695. | 5 |
| 711-715 | Female | Homemaker | College | \$700. | 5 |
| 716-720 | Male | Student | College | \$705. | 5 |
| 721-725 | Female | Homemaker | College | \$710. | 5 |
| 726-730 | Male | Student | College | \$715. | 5 |
| 731-735 | Female | Homemaker | College | \$720. | 5 |
| 736-740 | Male | Student | College | \$725. | 5 |
| 741-745 | Female | Homemaker | College | \$730. | 5 |
| 746-750 | Male | Student | College | \$735. | 5 |
| 751-755 | Female | Homemaker | College | \$740. | 5 |
| 756-760 | Male | Student | College | \$745. | 5 |
| 761-765 | Female | Homemaker | College | \$750. | 5 |
| 766-770 | Male | Student | College | \$755. | 5 |
| 771-775 | Female | Homemaker | College | \$760. | 5 |
| 776-780 | Male | Student | College | \$765. | 5 |
| 781-785 | Female | Homemaker | College | \$770. | 5 |
| 786-790 | Male | Student | College | \$775. | 5 |
| 791-795 | Female | Homemaker | College | \$780. | 5 |
| 796-800 | Male | Student | College | \$785. | 5 |
| 801-805 | Female | Homemaker | College | \$790. | 5 |
| 806-810 | Male | Student | College | \$795. | 5 |
| 811-815 | Female | Homemaker | College | \$800. | 5 |
| 816-820 | Male | Student | College | \$805. | 5 |
| 821-825 | Female | Homemaker | College | \$810. | 5 |
| 826-830 | Male | Student | College | \$815. | 5 |
| 831-835 | Female | Homemaker | College | \$820. | 5 |
| 836-840 | Male | Student | College | \$825. | 5 |
| 841-845 | Female | Homemaker | College | \$830. | 5 |
| 846-850 | Male | Student | College | \$835. | 5 |
| 851-855 | Female | Homemaker | College | \$840. | 5 |
| 856-860 | Male | Student | College | \$845. | 5 |
| 861-865 | Female | Homemaker | College | \$850. | 5 |
| 866-870 | Male | Student | College | \$855. | 5 |
| 871-875 | Female | Homemaker | College | \$860. | 5 |
| 876-880 | Male | Student | College | \$865. | 5 |
| 881-885 | Female | Homemaker | College | \$870. | 5 |
| 886-890 | Male | Student | College | \$875. | 5 |
| 891-895 | Female | Homemaker | College | \$880. | 5 |
| 896-900 | Male | Student | College | \$885. | 5 |
| 901-905 | Female | Homemaker | College | \$890. | 5 |
| 906-910 | Male | Student | College | \$895. | 5 |
| 911-915 | Female | Homemaker | College | \$900. | 5 |
| 916-920 | Male | Student | College | \$905. | 5 |
| 921-925 | Female | Homemaker | College | \$910. | 5 |
| 926-930 | Male | Student | College | \$915. | 5 |
| 931-935 | Female | Homemaker | College | \$920. | 5 |
| 936-940 | Male | Student | College | \$925. | 5 |
| 941-945 | Female | Homemaker | College | \$930. | 5 |
| 946-950 | Male | Student | College | \$935. | 5 |
| 951-955 | Female | Homemaker | College | \$940. | 5 |
| 956-960 | Male | Student | College | \$945. | 5 |
| 961-965 | Female | Homemaker | College | \$950. | 5 |
| 966-970 | Male | Student | College | \$955. | 5 |
| 971-975 | Female | Homemaker | College | \$960. | 5 |
| 976-980 | Male | Student | College | \$965. | 5 |
| 981-985 | Female | Homemaker | College | \$970. | 5 |
| 986-990 | Male | Student | College | \$975. | 5 |
| 991-995 | Female | Homemaker | College | \$980. | 5 |
| 996-1000 | Male | Student | College | \$985. | 5 |
| 1001-1005 | Female | Homemaker | College | \$990. | 5 |
| 1006-1010 | Male | Student | College | \$995. | 5 |
| 1011-1015 | Female | Homemaker | College | \$1000. | 5 |
| 1016-1020 | Male | Student | College | \$1005. | 5 |
| 1021-1025 | Female | Homemaker | College | \$1010. | 5 |
| 1026-1030 | Male | Student | College | \$1015. | 5 |
| 1031-1035 | Female | Homemaker | College | \$1020. | 5 |
| 1036-1040 | Male | Student | College | \$1025. | 5 |
| 1041-1045 | Female | Homemaker | College | \$1030. | 5 |
| 1046-1050 | Male | Student | College | \$1035. | 5 |
| 1051-1055 | Female | Homemaker | College | \$1040. | 5 |
| 1056-1060 | Male | Student | College | \$1045. | 5 |
| 1061-1065 | Female | Homemaker | College | \$1050. | 5 |
| 1066-1070 | Male | Student | College | \$1055. | 5 |
| 1071-1075 | Female | Homemaker | College | \$1060. | 5 |
| 1076-1080 | Male | Student | College | \$1065. | 5 |
| 1081-1085 | Female | Homemaker | College | \$1070. | 5 |
| 1086-1090 | Male | Student | College | \$1075. | 5 |
| 1091-1095 | Female | Homemaker | College | \$1080. | 5 |
| 1096-1100 | Male | Student | College | \$1085. | 5 |
| 1101-1105 | Female | Homemaker | College | \$1090. | 5 |
| 1106-1110 | Male | Student | College | \$1095. | 5 |
| 1111-1115 | Female | Homemaker | College | \$1100. | 5 |
| 1116-1120 | Male | Student | College | \$1105. | 5 |
| 1121-1125 | Female | Homemaker | College | \$1110. | 5 |
| 1126-1130 | Male | Student | College | \$1115. | 5 |
| 1131-1135 | Female | Homemaker | College | \$1120. | 5 |
| 1136-1140 | Male | Student | College | \$1125. | 5 |
| 1141-1145 | Female | Homemaker | College | \$1130. | 5 |
| 1146-1150 | Male | Student | College | \$1135. | 5 |
| 1151-1155 | Female | Homemaker | College | \$1140. | 5 |
| 1156-1160 | Male | Student | College | \$1145. | 5 |
| 1161-1165 | Female | Homemaker | College | \$1150. | 5 |
| 1166-1170 | Male | Student | College | \$1155. | 5 |
| 1171-1175 | Female | Homemaker | College | \$1160. | 5 |
| 1176-1180 | Male | Student | College | \$1165. | 5 |
| 1181-1185 | Female | Homemaker | College | \$1170. | 5 |
| 1186-1190 | Male | Student | College | \$1175. | 5 |
| 1191-1195 | Female | Homemaker | College | \$1180. | 5 |
| 1196-1200 | Male | Student | College | \$1185. | 5 |
| 1201-1205 | Female | Homemaker | College | \$1190. | 5 |
| 1206-1210 | Male | Student | College | \$1195. | 5 |
| 1211-1215 | Female</ | | | | |

TABLE IV

Evaluation of $R = 4(E_C - E_T)/(k^2 + 1/k^2 - 2)$ (Section VI)

Note: k and $(E_C - E_T)$ are evaluated in Table III

| <u>9</u> | <u>k^2</u> | <u>$1/k^2$</u> | <u>$(k^2 + 1/k^2 - 2)$</u> | <u>$1/4(k^2 + 1/k^2 - 2)$</u> | <u>$E_C - E_T$</u> | <u>R</u> |
|----------|-------------------------|---------------------------|---------------------------------------|--|-------------------------------|-----------------------|
| 2 | .99876 | 1.00124 | 0 | 0 | 0.047 | ∞ |
| 3 | .99720 | 1.00281 | .00001 | .0000025 | 0.107 | 42800 |
| 4 | .99503 | 1.004995 | .000025 | .000005 | 0.188 | 37600 |
| 5 | .99224 | 1.00782 | .00006 | .000015 | 0.295 | 19667 |
| 6 | .98884 | 1.01129 | .00013 | .000033 | 0.425 | 12878 |
| 7 | .98484 | 1.01539 | .00023 | .000058 | 0.577 | 9948 |
| 8 | .98024 | 1.02016 | .00040 | .000100 | 0.753 | 7530 |
| 9 | .97504 | 1.02560 | .00064 | .000160 | 0.950 | 5937 |
| 10 | .96925 | 1.03173 | .00098 | .000245 | 1.173 | 4788 |
| 11 | .96287 | 1.03856 | .00143 | .000358 | 1.416 | 3955 |
| 12 | .95592 | 1.04611 | .00203 | .000508 | 1.682 | 3311 |
| 13 | .94841 | 1.05440 | .00281 | .000703 | 1.968 | 2799 |
| 14 | .94033 | 1.06346 | .00379 | .000948 | 2.278 | 2402 |
| 16 | .92257 | 1.08393 | .00650 | .001625 | 2.957 | 1819 |
| 18 | .90273 | 1.10775 | .01048 | .002620 | 3.718 | 1419 |
| 20 | .88089 | 1.13522 | .01611 | .004028 | 4.556 | 1131 |
| 22 | .85718 | 1.16662 | .02380 | .005950 | 5.469 | 919 |
| 24 | .83172 | 1.20233 | .03405 | .008513 | 6.450 | 757 |
| 26 | .80462 | 1.24282 | .04744 | .011860 | 7.497 | 632 |
| 28 | .77605 | 1.28858 | .06463 | .016158 | 8.604 | 532 |
| 30 | .74613 | 1.34025 | .08638 | .021595 | 9.765 | 452 |

TABLE IV

Comparison of the results of the present work with those of other authors for the case of a uniform magnetic field.

| ω | $\frac{\omega}{\omega_c}$ | $\frac{\omega}{\omega_c} \left(\frac{1}{1 + \frac{\omega}{\omega_c}} \right)$ | $\frac{\omega}{\omega_c} \left(\frac{1}{1 + \frac{\omega}{\omega_c}} \right)^2$ | $\frac{\omega}{\omega_c} \left(\frac{1}{1 + \frac{\omega}{\omega_c}} \right)^3$ | $\frac{\omega}{\omega_c} \left(\frac{1}{1 + \frac{\omega}{\omega_c}} \right)^4$ | $\frac{\omega}{\omega_c} \left(\frac{1}{1 + \frac{\omega}{\omega_c}} \right)^5$ |
|----------|---------------------------|--|--|--|--|--|
| 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 1 | 0.5 | 0.333 | 0.111 | 0.037 | 0.012 | 0.004 |
| 2 | 0.333 | 0.200 | 0.067 | 0.022 | 0.007 | 0.002 |
| 3 | 0.250 | 0.167 | 0.056 | 0.019 | 0.006 | 0.002 |
| 4 | 0.200 | 0.143 | 0.048 | 0.016 | 0.005 | 0.001 |
| 5 | 0.167 | 0.125 | 0.042 | 0.014 | 0.004 | 0.001 |
| 6 | 0.143 | 0.111 | 0.037 | 0.012 | 0.004 | 0.001 |
| 7 | 0.125 | 0.100 | 0.033 | 0.011 | 0.003 | 0.001 |
| 8 | 0.111 | 0.091 | 0.030 | 0.010 | 0.003 | 0.001 |
| 9 | 0.100 | 0.083 | 0.028 | 0.009 | 0.003 | 0.001 |
| 10 | 0.091 | 0.077 | 0.026 | 0.008 | 0.002 | 0.001 |
| 15 | 0.067 | 0.059 | 0.020 | 0.006 | 0.002 | 0.000 |
| 20 | 0.050 | 0.048 | 0.016 | 0.005 | 0.001 | 0.000 |
| 25 | 0.040 | 0.038 | 0.013 | 0.004 | 0.001 | 0.000 |
| 30 | 0.033 | 0.030 | 0.010 | 0.003 | 0.001 | 0.000 |
| 35 | 0.029 | 0.027 | 0.009 | 0.003 | 0.001 | 0.000 |
| 40 | 0.025 | 0.024 | 0.008 | 0.002 | 0.001 | 0.000 |
| 45 | 0.022 | 0.022 | 0.007 | 0.002 | 0.001 | 0.000 |
| 50 | 0.020 | 0.020 | 0.006 | 0.002 | 0.001 | 0.000 |
| 55 | 0.018 | 0.018 | 0.006 | 0.002 | 0.001 | 0.000 |
| 60 | 0.017 | 0.017 | 0.005 | 0.002 | 0.001 | 0.000 |
| 65 | 0.015 | 0.015 | 0.005 | 0.002 | 0.001 | 0.000 |
| 70 | 0.014 | 0.014 | 0.004 | 0.002 | 0.001 | 0.000 |
| 75 | 0.013 | 0.013 | 0.004 | 0.002 | 0.001 | 0.000 |
| 80 | 0.012 | 0.012 | 0.004 | 0.002 | 0.001 | 0.000 |
| 85 | 0.011 | 0.011 | 0.003 | 0.002 | 0.001 | 0.000 |
| 90 | 0.011 | 0.011 | 0.003 | 0.002 | 0.001 | 0.000 |
| 95 | 0.010 | 0.010 | 0.003 | 0.002 | 0.001 | 0.000 |
| 100 | 0.010 | 0.010 | 0.003 | 0.002 | 0.001 | 0.000 |

TABLE V(A-G)

Evaluation of the correction term K and target thickness ΔE in Equation 14
for an effective counter half-angle θ_{eff} of 4-28 degrees

| θ_{eff} | $(E_p - E_T)$ | $\frac{(E_p - E_T)^{\frac{1}{2}}}{(E_p - E_c)^{\frac{1}{2}}}$ | $(E_p - E_c)$ | $\frac{(E_p - E_c)^{\frac{1}{2}}}{k(E_p - E_c)^{\frac{1}{2}}}$ | $\frac{1}{K^2}$ | K | ΔE | |
|----------------|---------------|---|---------------|--|-----------------|--------|------------|------|
| 4 | 2 | 1.41121 | 1.812 | 1.34617 | 1.34282 | .07139 | .005 | 2 |
| | 4 | 2.00000 | 3.812 | 1.95245 | 1.94759 | .05241 | .003 | 4 |
| | 8 | 2.82843 | 7.812 | 2.79500 | 2.78804 | .04039 | .002 | 8 |
| | 12 | 3.46410 | 11.812 | 3.43685 | 3.42829 | .03581 | .001 | 12 |
| | 16 | 4.00000 | 15.812 | 3.97644 | 3.96654 | .03346 | .001 | 16 |
| | 20 | 4.47214 | 19.812 | 4.45108 | 4.44000 | .03214 | .001 | 20 |
| 8 | 2 | 1.41121 | 1.247 | 1.11669 | 1.10560 | .30861 | .095 | 1.9 |
| | 4 | 2.00000 | 3.247 | 1.80193 | 1.78404 | .21596 | .047 | 4 |
| | 8 | 2.82843 | 7.247 | 2.69204 | 2.66531 | .16312 | .027 | 8 |
| | 12 | 3.46410 | 11.247 | 3.35365 | 3.32035 | .11375 | .021 | 12 |
| | 16 | 4.00000 | 15.247 | 3.90477 | 3.86600 | .13400 | .018 | 16 |
| | 20 | 4.47214 | 19.247 | 4.38712 | 4.34356 | .12858 | .017 | 20 |
| 12 | 2 | 1.41121 | 0.318 | .56392 | .55135 | .86286 | .745 | 1.3 |
| | 4 | 2.00000 | 2.318 | 1.52249 | 1.48855 | .51145 | .262 | 3.7 |
| | 8 | 2.82843 | 6.318 | 2.51355 | 2.45752 | .37091 | .138 | 7.9 |
| | 12 | 3.46410 | 10.318 | 3.21216 | 3.14056 | .32354 | .105 | 11.9 |
| | 16 | 4.00000 | 14.318 | 3.78388 | 3.69954 | .30046 | .090 | 15.9 |
| | 20 | 4.47214 | 18.318 | 4.27999 | 4.18459 | .28755 | .083 | 19.9 |

(1-A) V LIBAT

evaluation of the collection series and a brief description of the series.

for an effective counter, half-angle of 15-20 degrees

[illegible]

TABLE V(A-G)
(Continued)

| q_{eff} | $(E_p - E_T)$ | $(E_p - E_T)^{\frac{1}{2}}$ | $(E_p - E_C)$ | $(E_p - E_C)^{\frac{1}{2}}$ | $k(E_p - E_C)^{\frac{1}{2}}$ | $\frac{1}{K^2}$ | K | ΔE |
|------------------|---------------|-----------------------------|---------------|-----------------------------|------------------------------|-----------------|-------|------------|
| 16 | 4 | 2.00000 | 1.043 | 1.02127 | 0.98093 | 1.01907 | 1.039 | 3.0 |
| | 8 | 2.82843 | 5.043 | 2.24569 | 2.15699 | .671144 | .451 | 7.5 |
| | 12 | 3.46410 | 9.043 | 3.00717 | 2.88839 | .57571 | .331 | 11.7 |
| | 16 | 4.00000 | 13.043 | 3.611514 | 3.46886 | .531144 | .282 | 15.7 |
| | 20 | 4.47214 | 17.043 | 4.12832 | 3.96525 | .50689 | .257 | 19.7 |
| 20 | 5 | 2.23607 | 0.444 | .66633 | .62538 | 1.61069 | 2.594 | 2.4 |
| | 8 | 2.82843 | 3.444 | 1.85580 | 1.74176 | 1.08667 | 1.181 | 6.8 |
| | 12 | 3.46410 | 7.444 | 2.72836 | 2.56070 | .90340 | .816 | 11.2 |
| | 16 | 4.00000 | 11.444 | 3.38289 | 3.17501 | .82499 | .681 | 15.3 |
| | 20 | 4.47214 | 15.444 | 3.92984 | 3.68835 | .78379 | .614 | 19.4 |
| 24 | 8 | 2.82843 | 1.550 | 1.24495 | 1.13538 | 1.69305 | 2.867 | 5.1 |
| | 12 | 3.46410 | 5.550 | 2.35584 | 2.14850 | 1.31560 | 1.731 | 10.3 |
| | 16 | 4.00000 | 9.550 | 3.09033 | 2.81835 | 1.18165 | 1.396 | 14.6 |
| | 20 | 4.47214 | 13.550 | 3.68285 | 3.35872 | 1.11342 | 1.240 | 18.8 |
| 28 | 10 | 3.16278 | 1.396 | 1.18152 | 1.04085 | 2.12193 | 4.503 | 5.5 |
| | 12 | 3.46410 | 3.396 | 1.84282 | 1.62341 | 1.84069 | 3.388 | 8.6 |
| | 16 | 4.00000 | 7.396 | 2.71956 | 2.39577 | 1.60423 | 2.574 | 13.4 |
| | 20 | 4.47214 | 11.396 | 3.37580 | 2.97388 | 1.49826 | 2.245 | 17.8 |

| Year | Age | Sex | Length (cm) | Weight (kg) | Wing (cm) | Tail (cm) | Bill (cm) | Foot (cm) | Claw (cm) |
|------|-----|--------|-------------|-------------|-----------|-----------|-----------|-----------|-----------|
| 1960 | 10 | Male | 180.0 | 1.20 | 110.0 | 70.0 | 15.0 | 20.0 | 1.5 |
| 1961 | 11 | Female | 175.0 | 1.10 | 105.0 | 68.0 | 14.0 | 19.0 | 1.4 |
| 1962 | 12 | Male | 185.0 | 1.30 | 115.0 | 72.0 | 16.0 | 21.0 | 1.6 |
| 1963 | 13 | Female | 190.0 | 1.40 | 120.0 | 75.0 | 17.0 | 22.0 | 1.7 |
| 1964 | 14 | Male | 195.0 | 1.50 | 125.0 | 78.0 | 18.0 | 23.0 | 1.8 |
| 1965 | 15 | Female | 200.0 | 1.60 | 130.0 | 80.0 | 19.0 | 24.0 | 1.9 |
| 1966 | 16 | Male | 205.0 | 1.70 | 135.0 | 82.0 | 20.0 | 25.0 | 2.0 |
| 1967 | 17 | Female | 210.0 | 1.80 | 140.0 | 85.0 | 21.0 | 26.0 | 2.1 |
| 1968 | 18 | Male | 215.0 | 1.90 | 145.0 | 88.0 | 22.0 | 27.0 | 2.2 |
| 1969 | 19 | Female | 220.0 | 2.00 | 150.0 | 90.0 | 23.0 | 28.0 | 2.3 |
| 1970 | 20 | Male | 225.0 | 2.10 | 155.0 | 92.0 | 24.0 | 29.0 | 2.4 |
| 1971 | 21 | Female | 230.0 | 2.20 | 160.0 | 95.0 | 25.0 | 30.0 | 2.5 |
| 1972 | 22 | Male | 235.0 | 2.30 | 165.0 | 98.0 | 26.0 | 31.0 | 2.6 |
| 1973 | 23 | Female | 240.0 | 2.40 | 170.0 | 100.0 | 27.0 | 32.0 | 2.7 |
| 1974 | 24 | Male | 245.0 | 2.50 | 175.0 | 102.0 | 28.0 | 33.0 | 2.8 |
| 1975 | 25 | Female | 250.0 | 2.60 | 180.0 | 105.0 | 29.0 | 34.0 | 2.9 |
| 1976 | 26 | Male | 255.0 | 2.70 | 185.0 | 108.0 | 30.0 | 35.0 | 3.0 |
| 1977 | 27 | Female | 260.0 | 2.80 | 190.0 | 110.0 | 31.0 | 36.0 | 3.1 |
| 1978 | 28 | Male | 265.0 | 2.90 | 195.0 | 112.0 | 32.0 | 37.0 | 3.2 |
| 1979 | 29 | Female | 270.0 | 3.00 | 200.0 | 115.0 | 33.0 | 38.0 | 3.3 |
| 1980 | 30 | Male | 275.0 | 3.10 | 205.0 | 118.0 | 34.0 | 39.0 | 3.4 |

TABLE VI (A)

Evaluation of terms included in equation 30

| <u>θ</u> | <u>Cos θ</u> | <u>Sin² θ</u> | <u>Cos θ
Sin² θ</u> | <u>.0103
Cos θ
Sin² θ</u> | <u>.71850
Cos θ
Sin² θ</u> | <u>19.385
Cos θ
Sin² θ</u> |
|----------------------------|--------------------------------|--|--|--|---|---|
| 3 | .99863 | .00274 | .00274 | .00003 | .00197 | .05311 |
| 5 | .99619 | .00760 | .00757 | .00008 | .00546 | .14674 |
| 7 | .99255 | .01485 | .01474 | .00015 | .01099 | .28573 |
| 10 | .98481 | .03015 | .02969 | .00031 | .02199 | .57554 |
| 13 | .97437 | .05060 | .04930 | .00051 | .03636 | .95568 |
| 16 | .96126 | .07598 | .07304 | .00075 | .05459 | 1.41588 |
| 20 | .93969 | .11698 | .10992 | .00113 | .08405 | 2.13080 |
| 24 | .91355 | .16543 | .15113 | .00156 | .11886 | 2.92966 |

(A) TABLE VI

Comparison of test results with theoretical values

| Test No. | Test Result | Theoretical Value | Test No. | Test Result | Theoretical Value | Test No. |
|----------|-------------|-------------------|----------|-------------|-------------------|----------|
| 1001 | 1000 | 1000 | 1002 | 1000 | 1000 | 1 |
| 1003 | 1000 | 1000 | 1004 | 1000 | 1000 | 2 |
| 1005 | 1000 | 1000 | 1006 | 1000 | 1000 | 3 |
| 1007 | 1000 | 1000 | 1008 | 1000 | 1000 | 10 |
| 1009 | 1000 | 1000 | 1010 | 1000 | 1000 | 12 |
| 1011 | 1000 | 1000 | 1012 | 1000 | 1000 | 14 |
| 1013 | 1000 | 1000 | 1014 | 1000 | 1000 | 20 |
| 1015 | 1000 | 1000 | 1016 | 1000 | 1000 | 24 |

TABIE VI (B)

$$\ell_n 78 = 4.35671$$

| ΔE (kev) | $\ell_n(78 - \Delta E)$ | $\ell_n 78 - \ell_n(78 - \Delta E)$ |
|------------------|-------------------------|-------------------------------------|
| 1 | 4.34381 | .01290 |
| 2 | 4.33073 | .02598 |
| 4 | 4.30407 | .05264 |
| 8 | 4.24850 | .10821 |
| 12 | 4.18965 | .16706 |
| 16 | 4.12713 | .22958 |
| 20 | 4.06044 | .29627 |
| 24 | 3.98898 | .36773 |
| 28 | 3.91202 | .44469 |
| 32 | 3.82864 | .52807 |

TABIE VI (C)

| θ° | $(2/\sigma^2)N_c$ |
|----------------|---|
| 3 | .00337 ΔE + 0.05311 [$\ell_n 78 - \ell_n(78 - \Delta E)$] |
| 5 | .00935 ΔE + 0.14674 " |
| 7 | .01859 ΔE + 0.28573 " |
| 10 | .03749 ΔE + 0.57554 " |
| 13 | .06250 ΔE + 0.95568 " |
| 16 | .09408 ΔE + 1.41588 " |
| 20 | .14549 ΔE + 2.13080 " |
| 24 | .20687 ΔE + 2.92996 " |

Evaluation of terms appearing in equation 30

(7) 17 11847

17075.4 = 87.7

17075.4 = 87.7

17075.4 = 87.7

17075.4 = 87.7

17075.4

17075.4

1

17075.4

17075.4

2

17075.4

17075.4

3

17075.4

17075.4

4

17075.4

17075.4

5

17075.4

17075.4

6

17075.4

17075.4

7

17075.4

17075.4

8

17075.4

17075.4

9

17075.4

17075.4

10

(7) 17 11847

17075.4 = 87.7

17075.4 = 87.7

17075.4 = 87.7

17075.4 = 87.7

17075.4 = 87.7

17075.4 = 87.7

17075.4 = 87.7

17075.4 = 87.7

17075.4 = 87.7

TABLE VII (A)

Evaluation of equation 30 for a counter half-angle of 3 degrees

| ΔE | $.00337 \Delta E$ | $.05311 \ln(78/78-\Delta E)$ | $(2/\sigma Z)N_c$ | $N_c \propto$ | $(2/\sigma Z)N_c/\Delta E$ |
|------------|-------------------|------------------------------|-------------------|---------------|----------------------------|
| 1 | .00337 | .00069 | .00406 | 1.000 | .00406 |
| 2 | .00674 | .00138 | .00812 | 2.000 | .00406 |
| 4 | .01348 | .00280 | .01628 | 4.001 | .00407 |
| 8 | .02696 | .00575 | .03271 | 8.057 | .00409 |
| 12 | .04044 | .00873 | .04917 | 12.111 | .00410 |
| 16 | .05392 | .01219 | .06611 | 16.283 | .00413 |
| 20 | .06740 | .01573 | .08313 | 20.475 | .00416 |
| 24 | .08088 | .01953 | .10041 | 24.732 | .00418 |
| 28 | .09436 | .02362 | .11798 | 29.059 | .00421 |
| 30 | .10784 | .02805 | .13589 | 33.470 | .00425 |

TABLE VII (a)

Estimation of education SD for a counter-factual of 3 degrees

| Age | 00000000 | 00000000 | 00000000 | 00000000 |
|-----|----------|----------|----------|----------|
| 1 | 00000000 | 00000000 | 00000000 | 00000000 |
| 2 | 00000000 | 00000000 | 00000000 | 00000000 |
| 4 | 00000000 | 00000000 | 00000000 | 00000000 |
| 8 | 00000000 | 00000000 | 00000000 | 00000000 |
| 12 | 00000000 | 00000000 | 00000000 | 00000000 |
| 16 | 00000000 | 00000000 | 00000000 | 00000000 |
| 20 | 00000000 | 00000000 | 00000000 | 00000000 |
| 24 | 00000000 | 00000000 | 00000000 | 00000000 |
| 28 | 00000000 | 00000000 | 00000000 | 00000000 |
| 30 | 00000000 | 00000000 | 00000000 | 00000000 |

TABLE VII (B)

Evaluation of equation 30 for a counter half-angle of 5 degrees

| ΔE | $.00935\Delta E$ | $.14674 \ln(78/78-\Delta E)$ | $(2/\sigma Z)N_C$ | $N_C \propto$ | $(2/\sigma Z)N_C/\Delta E$ |
|------------|------------------|------------------------------|-------------------|---------------|----------------------------|
| 1 | .00935 | .00189 | .01124 | 1.0000 | .01124 |
| 2 | .01870 | .00381 | .02251 | 2.003 | .01126 |
| 4 | .03740 | .00772 | .04512 | 4.014 | .01128 |
| 8 | .07480 | .01588 | .09068 | 8.068 | .01134 |
| 12 | .11220 | .02451 | .13671 | 12.163 | .01139 |
| 16 | .14960 | .03369 | .18329 | 16.307 | .01146 |
| 20 | .18700 | .04347 | .23047 | 20.504 | .01152 |
| 24 | .22440 | .05396 | .27836 | 24.765 | .01160 |
| 28 | .26180 | .06525 | .32705 | 29.097 | .01168 |
| 32 | .29920 | .07749 | .37669 | 33.513 | .01177 |

TABLE VII (3)

Estimation of optimum for a counter, 100-100000

| 72 | 100000 | 10000 | 1000 | 100 | 10 |
|----|--------|--------|--------|--------|--------|
| 1 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| 2 | 0.0100 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| 3 | 0.0300 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| 4 | 0.0600 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| 5 | 0.0900 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| 10 | 0.1800 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| 20 | 0.3600 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| 30 | 0.5400 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| 40 | 0.7200 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| 50 | 0.9000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| 60 | 1.0800 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| 70 | 1.2600 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| 80 | 1.4400 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| 90 | 1.6200 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |

TABLE VII (C)

Evaluation of equation 30 for a counter half-angle of 7 degrees

| <u>ΔE</u> | <u>$.01859 \Delta E$</u> | <u>$.28573 \ln(78/78-\Delta E)$</u> | <u>$(2/\sigma-Z)N_C$</u> | <u>$N_C \propto$</u> | <u>$(2/\sigma-Z)N_C/\Delta E$</u> |
|------------------------------|-------------------------------------|--|-------------------------------------|---------------------------------|--|
| 1 | .01859 | .00369 | .02228 | 1.0000 | .02228 |
| 2 | .03718 | .00742 | .04460 | 2.002 | .02230 |
| 4 | .07436 | .01504 | .08940 | 4.013 | .02235 |
| 8 | .14872 | .03092 | .17964 | 8.063 | .02247 |
| 12 | .22308 | .04773 | .27081 | 12.155 | .02257 |
| 16 | .29744 | .06560 | .36304 | 16.294 | .02269 |
| 20 | .37180 | .08465 | .45645 | 20.487 | .02282 |
| 24 | .44616 | .10507 | .55123 | 24.741 | .02298 |
| 28 | .52052 | .12706 | .64758 | 29.066 | .02313 |
| 32 | .59488 | .15089 | .74577 | 33.473 | .02331 |

TABLE VII (O)

Expenditure of education for a number of years in the year 1950

| Year | 1950 | 1951 | 1952 | 1953 | 1954 | 1955 |
|------|--------|--------|--------|--------|--------|--------|
| 1 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 |
| 2 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 |
| 3 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 |
| 4 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 |
| 5 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 |
| 6 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 |
| 7 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 |
| 8 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 |
| 9 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 |
| 10 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 |
| 11 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 |
| 12 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 |
| 13 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 |
| 14 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 |
| 15 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 |
| 16 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 |
| 17 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 |
| 18 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 |
| 19 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 |
| 20 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 |

TABLE VII (D)

Evaluation of equation 30 for a counter half-angle of 10 degrees

| ΔE | $.03749 \Delta E$ | $.57554 \ln(78/78-\Delta E)$ | $(2/\sigma-Z)N_c$ | $N_c \propto$ | $(2/\sigma-Z)N_c/\Delta E$ |
|------------|-------------------|------------------------------|-------------------|---------------|----------------------------|
| 1 | .03749 | .00742 | .04491 | 1.0000 | .04491 |
| 2 | .07498 | .01495 | .08993 | 2.002 | .04497 |
| 4 | .14996 | .03030 | .18026 | 4.014 | .04507 |
| 8 | .29992 | .06228 | .36220 | 8.065 | .04528 |
| 12 | .44988 | .09615 | .54603 | 12.158 | .04550 |
| 16 | .59984 | .13213 | .73197 | 16.299 | .04575 |
| 20 | .74980 | .17052 | .92032 | 20.493 | .04602 |
| 24 | .89976 | .21164 | 1.11140 | 24.747 | .04631 |
| 28 | 1.04972 | .25594 | 1.30566 | 29.073 | .04663 |
| 32 | 1.19968 | .30393 | 1.50361 | 33.481 | .04699 |

TABLE VII (E)

Evaluation of equation 30 for a counter half-angle of 13 degrees

| ΔE | $.06250 \Delta E$ | $.95568 \ln(78/78-\Delta E)$ | $(2/\sigma-Z)N_c$ | $N_c \propto$ | $(2/\sigma-Z)N_c/\Delta E$ |
|------------|-------------------|------------------------------|-------------------|---------------|----------------------------|
| 1 | .06250 | .01233 | .07483 | 1.0000 | .07483 |
| 2 | .12500 | .02483 | .14983 | 2.002 | .07492 |
| 4 | .25000 | .05031 | .30031 | 4.013 | .07508 |
| 8 | .50000 | .10341 | .60341 | 8.064 | .07543 |
| 12 | .75000 | .15966 | .90966 | 12.156 | .07581 |
| 16 | 1.00000 | .21941 | 1.21941 | 16.296 | .07621 |
| 20 | 1.25000 | .28314 | 1.53314 | 20.488 | .07666 |
| 24 | 1.50000 | .35143 | 1.85143 | 24.742 | .07714 |
| 28 | 1.75000 | .42498 | 2.17498 | 29.066 | .07768 |
| 32 | 2.00000 | .50467 | 2.50467 | 33.471 | .07827 |

TABLE VII (N)

Evaluation of equation 10 for a constant value of β between

| β | $\beta = 0.0250$ | $\beta = 0.0500$ | $\beta = 0.0750$ | $\beta = 0.1000$ | $\beta = 0.1250$ |
|---------|------------------|------------------|------------------|------------------|------------------|
| 1 | 0.0250 | 0.0500 | 0.0750 | 0.1000 | 0.1250 |
| 2 | 0.0500 | 0.1000 | 0.1500 | 0.2000 | 0.2500 |
| 3 | 0.0750 | 0.1500 | 0.2250 | 0.3000 | 0.3750 |
| 4 | 0.1000 | 0.2000 | 0.3000 | 0.4000 | 0.5000 |
| 5 | 0.1250 | 0.2500 | 0.3750 | 0.5000 | 0.6250 |
| 6 | 0.1500 | 0.3000 | 0.4500 | 0.6000 | 0.7500 |
| 7 | 0.1750 | 0.3500 | 0.5250 | 0.7000 | 0.8750 |
| 8 | 0.2000 | 0.4000 | 0.6000 | 0.8000 | 1.0000 |
| 9 | 0.2250 | 0.4500 | 0.6750 | 0.9000 | 1.1250 |
| 10 | 0.2500 | 0.5000 | 0.7500 | 1.0000 | 1.2500 |
| 11 | 0.2750 | 0.5500 | 0.8250 | 1.1000 | 1.3750 |
| 12 | 0.3000 | 0.6000 | 0.9000 | 1.2000 | 1.5000 |
| 13 | 0.3250 | 0.6500 | 0.9750 | 1.3000 | 1.6250 |
| 14 | 0.3500 | 0.7000 | 1.0500 | 1.4000 | 1.7500 |
| 15 | 0.3750 | 0.7500 | 1.1250 | 1.5000 | 1.8750 |
| 16 | 0.4000 | 0.8000 | 1.2000 | 1.6000 | 2.0000 |
| 17 | 0.4250 | 0.8500 | 1.2750 | 1.7000 | 2.1250 |
| 18 | 0.4500 | 0.9000 | 1.3500 | 1.8000 | 2.2500 |
| 19 | 0.4750 | 0.9500 | 1.4250 | 1.9000 | 2.3750 |
| 20 | 0.5000 | 1.0000 | 1.5000 | 2.0000 | 2.5000 |

TABLE VII (F)

Evaluation of equation 30 for a counter half-angle of 16 degrees

| ΔE | $.09408 \Delta E$ | $1.41588 \ln(78/78-\Delta E)$ | $(2/\sigma Z)N_C$ | $N_C \propto$ | $(2/\sigma Z)N_C/\Delta E$ |
|------------|-------------------|-------------------------------|-------------------|---------------|----------------------------|
| 1 | .09408 | .01826 | .11234 | 1.0000 | .11234 |
| 2 | .18816 | .03678 | .22494 | 2.002 | .11247 |
| 4 | .37632 | .07453 | .45085 | 4.013 | .11271 |
| 8 | .75264 | .15321 | .90585 | 8.063 | .11323 |
| 12 | 1.12896 | .23654 | 1.36550 | 12.156 | .11379 |
| 16 | 1.50528 | .32506 | 1.83034 | 16.293 | .11439 |
| 20 | 1.88160 | .41948 | 2.30108 | 20.483 | .11505 |
| 24 | 2.25792 | .52066 | 2.77858 | 24.734 | .11577 |
| 28 | 2.63424 | .62963 | 3.26387 | 29.053 | .11657 |
| 32 | 3.01056 | .74768 | 3.75824 | 33.454 | .11745 |

(c) IIV ZIAT

creases in the amount of time spent on the job by the employees.

[illegible]

TABLE VII (G)

Evaluation of equation 30 for a counter half-angle of 20 degrees

| <u>ΔE</u> | <u>.14549 ΔE</u> | <u>2.13080 $\ln(78/78-\Delta E)$</u> | <u>$(2/\sigma Z)N_c$</u> | <u>$N_c \propto$</u> | <u>$(2/\sigma Z)N_c/\Delta E_c$</u> |
|------------------------------|-------------------------------------|---|-------------------------------------|---------------------------------|--|
| 1 | .14549 | .02749 | .17298 | 1.0000 | .17298 |
| 2 | .29098 | .05536 | .34634 | 2.002 | .17317 |
| 4 | .58196 | .11217 | .69413 | 4.013 | .17353 |
| 8 | 1.16392 | .23057 | 1.39449 | 8.062 | .17431 |
| 12 | 1.74588 | .35597 | 2.10185 | 12.151 | .17515 |
| 16 | 2.32784 | .48919 | 2.81703 | 16.283 | .17606 |
| 20 | 2.90980 | .63129 | 3.54109 | 20.471 | .17705 |
| 24 | 3.49176 | .78356 | 4.27532 | 24.716 | .17814 |
| 28 | 4.07372 | .94754 | 5.02126 | 29.028 | .17933 |
| 32 | 4.65568 | 1.12521 | 5.78089 | 33.419 | .18065 |

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Examination of evidence for a possible connection between the two cases.

| Case No. | Date of Birth | Place of Birth | Occupation | Address | Remarks |
|----------|---------------|----------------|------------|---------|---------|
| 100-1 | 1901.1 | 1901.1 | 1901.1 | 1901.1 | 1 |
| 100-2 | 1901.2 | 1901.2 | 1901.2 | 1901.2 | 2 |
| 100-3 | 1901.3 | 1901.3 | 1901.3 | 1901.3 | 3 |
| 100-4 | 1901.4 | 1901.4 | 1901.4 | 1901.4 | 4 |
| 100-5 | 1901.5 | 1901.5 | 1901.5 | 1901.5 | 5 |
| 100-6 | 1901.6 | 1901.6 | 1901.6 | 1901.6 | 6 |
| 100-7 | 1901.7 | 1901.7 | 1901.7 | 1901.7 | 7 |
| 100-8 | 1901.8 | 1901.8 | 1901.8 | 1901.8 | 8 |
| 100-9 | 1901.9 | 1901.9 | 1901.9 | 1901.9 | 9 |
| 100-10 | 1901.10 | 1901.10 | 1901.10 | 1901.10 | 10 |
| 100-11 | 1901.11 | 1901.11 | 1901.11 | 1901.11 | 11 |
| 100-12 | 1901.12 | 1901.12 | 1901.12 | 1901.12 | 12 |
| 100-13 | 1901.13 | 1901.13 | 1901.13 | 1901.13 | 13 |
| 100-14 | 1901.14 | 1901.14 | 1901.14 | 1901.14 | 14 |
| 100-15 | 1901.15 | 1901.15 | 1901.15 | 1901.15 | 15 |
| 100-16 | 1901.16 | 1901.16 | 1901.16 | 1901.16 | 16 |
| 100-17 | 1901.17 | 1901.17 | 1901.17 | 1901.17 | 17 |
| 100-18 | 1901.18 | 1901.18 | 1901.18 | 1901.18 | 18 |
| 100-19 | 1901.19 | 1901.19 | 1901.19 | 1901.19 | 19 |
| 100-20 | 1901.20 | 1901.20 | 1901.20 | 1901.20 | 20 |

TABLE VII (H)

Evaluation of equation 30 for a counter half-angle of 24 degrees

| ΔE | $.20687 \Delta E$ | $2.92996 \ln(78/78-\Delta E)$ | $(2/\sigma Z)N_c$ | $N_c \alpha$ | $(2/\sigma Z)N_c/\Delta E$ |
|------------|-------------------|-------------------------------|-------------------|--------------|----------------------------|
| 1 | .20687 | .03780 | .24467 | 1.0000 | .24467 |
| 2 | .41374 | .07612 | .48986 | 2.002 | .24493 |
| 4 | .82748 | .15423 | .98171 | 4.013 | .24543 |
| 8 | 1.65496 | .31704 | 1.97200 | 8.060 | .24650 |
| 12 | 2.48244 | .48947 | 2.97191 | 12.147 | .24766 |
| 16 | 3.30992 | .67265 | 3.98257 | 16.277 | .24891 |
| 20 | 4.13740 | .86804 | 5.00544 | 20.458 | .25027 |
| 24 | 4.96488 | 1.07741 | 6.04229 | 24.696 | .25176 |
| 28 | 5.79236 | 1.30290 | 7.09526 | 28.999 | .25340 |
| 32 | 6.61984 | 1.54719 | 8.16703 | 33.380 | .25522 |

TABLE 1 (continued)

Evaluation of equation 20 for a constant half-width of 1.0 mm

| Δt | Δx | Δy | Δz | Δw | Δv |
|------------|------------|------------|------------|------------|------------|
| 1 | 0.0001 | 0.0001 | 0.0001 | 0.0001 | 0.0001 |
| 2 | 0.0002 | 0.0002 | 0.0002 | 0.0002 | 0.0002 |
| 3 | 0.0003 | 0.0003 | 0.0003 | 0.0003 | 0.0003 |
| 4 | 0.0004 | 0.0004 | 0.0004 | 0.0004 | 0.0004 |
| 5 | 0.0005 | 0.0005 | 0.0005 | 0.0005 | 0.0005 |
| 6 | 0.0006 | 0.0006 | 0.0006 | 0.0006 | 0.0006 |
| 7 | 0.0007 | 0.0007 | 0.0007 | 0.0007 | 0.0007 |
| 8 | 0.0008 | 0.0008 | 0.0008 | 0.0008 | 0.0008 |
| 9 | 0.0009 | 0.0009 | 0.0009 | 0.0009 | 0.0009 |
| 10 | 0.0010 | 0.0010 | 0.0010 | 0.0010 | 0.0010 |
| 11 | 0.0011 | 0.0011 | 0.0011 | 0.0011 | 0.0011 |
| 12 | 0.0012 | 0.0012 | 0.0012 | 0.0012 | 0.0012 |
| 13 | 0.0013 | 0.0013 | 0.0013 | 0.0013 | 0.0013 |
| 14 | 0.0014 | 0.0014 | 0.0014 | 0.0014 | 0.0014 |
| 15 | 0.0015 | 0.0015 | 0.0015 | 0.0015 | 0.0015 |

TABLE VIII

Theoretical relative counting rates of the long counter per unit target thickness at 1960-kev proton energy for various target thicknesses and various counter positions (fresh targets only). Normalized to unity at $\theta = 5$ degrees.

| $\frac{\Delta E}{\theta_{eff}}$ | 3 | 5 | 7 | 10 | 13 | 16 | 20 | 24 |
|---------------------------------|------|---|-------|-------|-------|-------|--------|--------|
| 1 | .361 | 1 | 1.982 | 3.996 | 6.657 | 9.995 | 15.390 | 21.768 |
| 2 | .361 | 1 | 1.980 | 3.994 | 6.654 | 9.988 | 15.379 | 21.752 |
| 4 | .361 | 1 | 1.981 | 3.996 | 6.656 | 9.992 | 15.384 | 21.796 |
| 8 | .361 | 1 | 1.981 | 3.993 | 6.652 | 9.985 | 15.371 | 21.737 |
| 12 | .360 | 1 | 1.982 | 3.995 | 6.656 | 9.990 | 15.377 | 21.744 |
| 16 | .360 | 1 | 1.980 | 3.992 | 6.650 | 9.982 | 15.363 | 21.720 |
| 20 | .361 | 1 | 1.981 | 3.995 | 6.655 | 9.987 | 15.369 | 21.725 |
| 24 | .360 | 1 | 1.981 | 3.992 | 6.650 | 9.980 | 15.357 | 21.703 |
| 28 | .360 | 1 | 1.980 | 3.992 | 6.651 | 9.980 | 15.354 | 21.695 |
| 32 | .361 | 1 | 1.980 | 3.992 | 6.650 | 9.979 | 15.348 | 21.684 |

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Theory of the relative motion of the two bodies is
 based on the fact that the two bodies are
 separated by a distance of 1000 miles and
 the distance between the two bodies is
 1000 miles.

[illegible]

APPENDIX C

POSSIBLE SEQUENCES OF CERTAIN DEFINED VALUES OF THE PROTON ENERGY WHICH OCCUR WHEN INTEGRATING OVER THE NEUTRON YIELD CURVE

As the proton energy increases, it passes in succession through the values of E_T , E_C , E_L , $E_T + \Delta E$, $E_C + \Delta E$, and $E_L + \Delta E$ in one of the five possible ways listed below:

| <u>I</u> | <u>II</u> | <u>III</u> | <u>IV</u> | <u>V</u> |
|------------------|------------------|------------------|------------------|------------------|
| E_T | E_T | E_T | E_T | E_T |
| E_C | $E_T + \Delta E$ | E_C | $E_T + \Delta E$ | E_C |
| $E_T + \Delta E$ | E_C | E_L | E_C | $E_T + \Delta E$ |
| $E_C + \Delta E$ | $E_C + \Delta E$ | $E_T + \Delta E$ | E_L | E_L |
| E_L | E_L | $E_C + \Delta E$ | $E_C + \Delta E$ | $E_C + \Delta E$ |
| $E_L + \Delta E$ | $E_L + \Delta E$ | $E_L + \Delta E$ | $E_L + \Delta E$ | $E_L + \Delta E$ |

The inequalities represented in each sequence will tell us under what conditions a particular sequence is applicable.

APPENDIX

TABLE I. - SUMMARY OF THE DATA OBTAINED FROM THE EXPERIMENT.

THE DATA ARE GIVEN IN THE FOLLOWING TABLES.

TABLE II. - DATA FOR THE FIRST EXPERIMENT.

TABLE III. - DATA FOR THE SECOND EXPERIMENT.

TABLE IV. - DATA FOR THE THIRD EXPERIMENT.

| Time (sec) | Distance (cm) | Velocity (cm/sec) | Acceleration (cm/sec ²) |
|------------|---------------|-------------------|-------------------------------------|
| 0 | 0 | 0 | 0 |
| 1 | 10 | 10 | 10 |
| 2 | 40 | 20 | 20 |
| 3 | 90 | 30 | 30 |
| 4 | 160 | 40 | 40 |
| 5 | 250 | 50 | 50 |
| 6 | 360 | 60 | 60 |
| 7 | 490 | 70 | 70 |
| 8 | 640 | 80 | 80 |
| 9 | 810 | 90 | 90 |
| 10 | 1000 | 100 | 100 |

TABLE V. - DATA FOR THE FOURTH EXPERIMENT.

TABLE VI. - DATA FOR THE FIFTH EXPERIMENT.

Case I:

$$\left. \begin{array}{l} \Delta E < E_L - E_C \\ \Delta E > E_C - E_T \end{array} \right\} \text{ or } \left. \begin{array}{l} E_C < E_L - \Delta E \\ E_C < E_T + \Delta E \end{array} \right\}$$

Therefore,

$$E_L - E_C + \Delta E > \Delta E + E_C - E_T$$

$$E_C < \left(\frac{E_L + E_T}{2} \right).$$

For $\text{Li}(p,n)$, this gives $E_C < (E_T + 19.85 \text{ kev})$.

This limitation on E_C is attained by keeping the counter half-angle less than 45 degrees. However, there is a more strict limitation upon E_C , so that we get:

$$E_C < (E_T + \Delta E) \quad \text{for } \Delta E < \left(\frac{E_L + E_T}{2} \right) = 19.85 \text{ kev.}$$

$$E_C < (E_L - \Delta E) \quad \text{for } \Delta E > \left(\frac{E_L + E_T}{2} \right) = 19.85 \text{ kev.}$$

Case II:

$$\left. \begin{array}{l} \Delta E < E_C - E_T \\ \Delta E < E_L - E_C \end{array} \right\} \text{ or } \left. \begin{array}{l} E_C > E_T + \Delta E \\ E_C < E_L - \Delta E \end{array} \right\}$$

$$E_C - E_T + E_L - E_C > 2\Delta E$$

$$\Delta E < \left(\frac{E_L - E_T}{2} \right) = 19.85 \text{ kev.}$$

This is the case for $\theta = 45^\circ$, at which angle $E_C = E_T + 19.85 = E_L - 19.85$ kev. For $\theta < 45^\circ$, $E_C < (E_T + 19.85)$ and $\Delta E < (E_C - E_T)$. For $\theta > 45^\circ$, $E_C > (E_T + 19.85)$ and $\Delta E < (E_L - E_C)$.

Case III:

$$\Delta E > (E_L - E_T) = 39.7 \text{ kev for } \text{Li}(p,n) \text{ reaction.}$$

Case IV:

$$\begin{array}{l} \Delta E < E_C - E_T \\ \Delta E > E_L - E_C \end{array} \quad \text{or} \quad \begin{array}{l} E_C > E_T + \Delta E \\ E_C > E_L - \Delta E \end{array}$$

$$E_C - E_T + \Delta E > \Delta E + E_L - E_C$$

$$E_C > \left(\frac{E_L + E_T}{2} \right) = (E_T + 19.85 \text{ kev}).$$

For $\Delta E = \left(\frac{E_L - E_T}{2} \right) = 19.85 \text{ kev}$, $E_C = E_T + 19.85 \text{ kev}$, and $\theta = 45^\circ$.

For $\Delta E > \left(\frac{E_L - E_T}{2} \right) = 19.85 \text{ kev}$, $E_C > (E_T + \Delta E)$, and $\theta > 45^\circ$.

For $\Delta E < \left(\frac{E_L - E_T}{2} \right) = 19.85 \text{ kev}$, $E_C > (E_L - \Delta E)$, and $\theta > 45^\circ$.

Thus, this case cannot occur for $\theta < 45^\circ$.

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$$\frac{1}{2} < \frac{1}{2} - \frac{1}{2} + \dots$$

$$\frac{1}{2} < \frac{1}{2} - \frac{1}{2} + \dots$$

Case V:

$$\begin{cases} (\Delta E < (E_L - E_T) = 39.7 \text{ kev for Li(p,n)} \\ (\Delta E > (E_C - E_T) \\ (\Delta E > (E_L - E_C) \end{cases}$$

Therefore,

$$2\Delta E > (E_L - E_T) = 39.7 \text{ kev.}$$

$$\Delta E > \left(\frac{E_L - E_T}{2} \right) = 19.85 \text{ kev.}$$

This is the smallest value of ΔE which may be measured in Case V,

and it occurs for $E_C = \left(\frac{E_L + E_T}{2} \right) = E_T + 19.85 \text{ kev}$, which corre-

sponds to $\theta = 45^\circ$.

Therefore,

$$\begin{aligned} \text{For } \theta = 45^\circ \quad & (E_C - E_T) < \Delta E < (E_L - E_T) \\ & 19.85 \text{ kev} < \Delta E < 39.7 \text{ kev.} \end{aligned}$$

$$\begin{aligned} \text{For } \theta < 45^\circ \quad & (E_L - E_C) < \Delta E < (E_L - E_T) \\ & (E_L - E_C) < \Delta E < 39.7 \text{ kev} \end{aligned}$$

where $(E_L - E_C) > 19.85 \text{ kev}$.

$$\text{For } \theta < 30^\circ \quad 30 \text{ kev} \leq \Delta E < 39.7 \text{ kev.}$$

$$\text{For } \theta = 5^\circ \quad 39.4 < \Delta E < 39.7 \text{ kev.}$$

Case V:

$$\begin{cases} (A_1 > (E_1 - E_2)) \\ (A_2 > (E_2 - E_3)) \\ (A_3 > (E_3 - E_4)) \end{cases}$$

Therefore,

$$A_1 > (E_1 - E_2) = 30.7 \text{ sec.}$$

$$A_2 > \frac{(E_1 - E_2)}{2} = 15.35 \text{ sec.}$$

This is the smallest value of A_2 which can be sustained in Case V, and it occurs for $E_1 = 1.0 \text{ sec.}$ and $E_2 = 0.7 \text{ sec.}$ which occurs

according to $\theta = 180^\circ$.

Therefore,

$$A_1 > 30.7 \text{ sec.} \quad \text{for } \theta = 180^\circ$$

$$A_2 > 15.35 \text{ sec.}$$

$$A_3 > 15.35 \text{ sec.} \quad \text{for } \theta = 180^\circ$$

$$A_4 > 15.35 \text{ sec.}$$

where $\theta = 180^\circ$ and $E_1 = 1.0 \text{ sec.}$

$$A_5 > 15.35 \text{ sec.}$$

$$A_6 > 15.35 \text{ sec.}$$

Summary:

- (a) For $\theta < 45^\circ$ and $\Delta E < 19.85$ kev only Cases I and II are possible.
- (b) For $\theta < 45^\circ$ and $19.85 < \Delta E < (E_L - E_C)$ only Case I is possible
- (c) For $\theta < 45^\circ$ and $19.85 < (E_L - E_C) < \Delta E$ only Cases I and V are possible.
- (d) For $\theta < 30^\circ$ and $\Delta E < 30$ kev, only cases I and II are possible.

Summary:

- (a) For $\theta < 150$ and $15 < 12.82$ no only Case I and II are possible.
- (b) For $\theta < 150$ and $12.82 < 11 < (11 - 10)$ only Case I is possible.
- (c) For $\theta < 150$ and $10.82 < (11 - 10) < 10$ only Case I and V are possible.
- (d) For $\theta < 300$ and $15 < 30$ no only Case I and II are possible.

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Thesis

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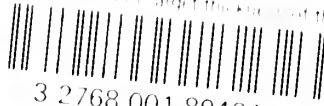
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